Half Wave Plates

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AMERICAN JOURNAL of PHYSICS

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Elementary Theory of Neutron Sources in Reactors*

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(Received July 22, 1952)

The basic physical and mathematical ideas involved in the use of a neutron source in reactors in the steady state are developed from the diffusion theory of neutrons of one energy group. The ideas are fundamental to more general and difficult discussions in published material and should serve as a helpful preliminary to the formal study of reactor theory.

1. STATEMENT OF PROBLEM

ONE of the important features generally not discussed about the operation of a nuclear chain reactor is the use of a strong source of neutrons other than fissionable material inside the reactor. It is the object of this article to point out the physical and engineering necessity for such an independent neutron source and to give a clear mathematical treatment of the role played by a neutron source for didactic purposes. The mathematical treatment is restricted by the simplifications of diffusion theory and monoenergetic neutrons in the reactor. However, in spite of these assumptions, the principal physical and mathematical features of the reactor in the steady state can be discussed in a straightforward way. This treatment should serve as a helpful preliminary to formal study of reactor theory.

In the steady state, four processes contribute to the maintenance of a desired level of activity or power output of a reactor. First, if we let $n(\mathbf{r})$ denote the neutron density at point $\mathbf{r} = (x, y, z)$, then from Fick's law the net diffusion rate of the neutrons out of an infinitesimal volume element dV is

$$-D\,\nabla^2 n\,dV,$$

where D is the diffusion coefficient, assumed to be position independent. This represents the effect of leakage. Second, neutrons are lost by capture in nuclei which do not undergo fission. This absorption effect is proportional to the neutron density, neutron speed, and inversely proportional to the mean free path for absorption. The rate of loss of neutrons is, therefore,

$$(nv/\lambda_c)\,dV,$$

where v is neutron speed and λ_c is total capture mean free path. Third, the rate of neutron creation is similarly expressed as

$$k_\infty\,(nv/\lambda_c)\,dV,$$

where k_∞ is the average number of neutrons produced per capture. This includes any type of capture loss. Fourth, neutrons may be supplied at a rate $\mathbf{R}dV$ by a source density distribution $\mathbf{R}(\mathbf{r})$ independent of the neutron density. The

* This paper is based upon ORNL-1014, declassified July 5, 1951, and prepared while the author served as Research Participant at Oak Ridge National Laboratory during the summer of 1950.

steady state is then characterized by the equation

$$-D\,\nabla^2 n+nv/\lambda_c=k_\infty\,(nv/\lambda_c)+\mathbf{R},$$

which, by rearrangement, takes the form

$$\nabla^2 n(\mathbf{r})+B^2\,n(\mathbf{r})+R(\mathbf{r})=0,$$

where

$$R=\mathbf{R}/D \quad \text{and} \quad B^2=(k_\infty-1)\,(v/D\,\lambda_c).$$

B^2 is called the "material buckling" or "material Laplacian" in contrast to a "geometrical buckling or Laplacian" due to reactor geometry.

The neutron density in the reactor is determined as the solution of a boundary value problem formulated as follows:

$$\left.\begin{aligned}&\nabla^2 n(\mathbf{r})+B^2\,n(\mathbf{r})+R(\mathbf{r})=0;\\&n(\mathbf{r})\geqslant 0,\quad n(\mathbf{r}_S)=0,\quad B^2>0,\quad R(\mathbf{r})\geqslant 0.\end{aligned}\right\} \quad (1)$$

$n(\mathbf{r}_S)$ represents the neutron density on the (extrapolated) boundary or bounding surface S which is designated by the position vector $\mathbf{r}_S$.

2. METHOD OF SOLUTION

The neutron balance equation in (1),

$$\nabla^2 n+B^2 n+R=0,$$

is an *inhomogeneous* equation which is compatible with the boundary condition

$$n(\mathbf{r}_S)=0,$$

that is to say, the solution of the balance equation exists and is finite and vanishes on S, provided certain conditions are placed on B^2 and R. We must take R to be *nonnegative* definite if it is to represent a *source* density and not a *sink* density anywhere in the reactor. As for B^2 it must be a *positive definite* constant if the reactor is to show any neutron multiplication, that is, if the probability for absorption producing fission is large enough to overcome nonfissionable absorption of any kind. The central problem is therefore to determine just what range of values B^2 must have in the practical case of operating a reactor. To do this we find the general representation of the solution for the problem (1) in what follows.

Consider first the problem for the critical reactor (no sources):

$$\left.\begin{aligned}&\nabla^2\psi(\mathbf{r})+B^2\psi(\mathbf{r})=0;\\&\psi(\mathbf{r})\not\equiv 0,\quad \psi(\mathbf{r}_S)=0,\quad B^2>0,\quad \psi(\mathbf{r})\geqslant 0,\end{aligned}\right\} \quad (2)$$

defined over the same reactor as Eq. (1). We know that associated with this problem there are a countably infinite number of positive discrete values of B^2 (eigenvalues) which we designate as $\{B_k^2\}$ (and which we can arrange in ascending order of magnitude: $0<B_1^2<B_2^2<\cdots<B_k^2<\cdots$) corresponding to which there is the sequence of linearly independent functions (eigenfunctions) $\{\psi_k\}$ such that for each B_k, ψ_k a weaker problem is solved:

$$\left.\begin{aligned}&\nabla^2\psi_k(\mathbf{r})+B_k^2\,\psi_k(\mathbf{r})=0;\\&\psi_k(\mathbf{r}_S)=0,\quad \psi_k(\mathbf{r})\not\equiv 0,\quad B_k^2>0.\end{aligned}\right\} \quad (3)$$

However, of all the solutions of the sequence $\{\psi_k\}$, only ψ_1, corresponding to the smallest of the sequence $\{B_k^2\}$, namely B_1^2, according to our arrangement, *does not change sign* anywhere *inside* the (extrapolated) boundary S of the region defined by the reactor. (The proof that such a positive definite solution in the interior exists for arbitrary shapes is a mathematical problem which we shall assume and may be found in mathematical treatises.) Notice further, that, because problem (3) is completely *homogeneous*, each solution ψ_k may be multiplied by an arbitrary constant, positive or negative, and still remain a perfectly valid mathematical solution. Physically, however, the only acceptable solution can be a nonnegative solution for the neutron density. Consequently, we take as the only physically possible solution for Eq. (2) the function

$$c\,\Psi_1(\mathbf{r})\equiv c\,\Psi(B_1^2,\mathbf{r})=c\,\frac{\psi_1(\mathbf{r})}{\left|\left(\int_{VS}\psi_1^2(\mathbf{r})\,dV\right)^{\frac{1}{2}}\right|}, \quad (4)$$

where Ψ_1 is in normalized form and c is an arbitrary nonnegative constant.

The constant B_1^2 is called the *geometrical buckling* or *geometrical Laplacian* mentioned above. It is the only value of B^2, other than zero, which can be used in the homogeneous equation of Eq. (2) so that the solution can fit the conditions of vanishing on the boundary and being nonnegative in the interior.

We shall call $\Psi(B_1^2,\mathbf{r})$ the *fundamental solution*. Similarly, we normalize all other mathe-

matical solutions of the sequence by writing

$$\Psi_k(\mathbf{r}) \equiv \Psi(B_k^2, \mathbf{r}) = \psi_k(\mathbf{r}) \left| \left(\int_{VS} \psi_k^2(\mathbf{r})\, dV \right)^{\frac{1}{2}} \right|^{-1}.$$

Hence, the sequence $\{\Psi_k\}$ constitutes an orthonormal set of functions in terms of which any piecewise continuous function may be represented in the Fourier manner:

$$f(\mathbf{r}) = \sum_{j=1}^{\infty} \Psi_j(\mathbf{r}) \int_{VS} f(\mathbf{r}')\, \Psi_j(\mathbf{r}')\, dV'. \qquad (5)$$

Consider next the original problem (1). In accordance with Eq. (5), let us assume

$$n = \sum_{j=1}^{\infty} b_j\, \Psi_j,\ R = \sum_{j=1}^{\infty} \Psi_j \int_{VS} \Psi_j R dV.$$

We find from Eq. (1) that

$$\sum_{j=1}^{\infty} \left[b_j(B^2 - B_j^2) + \int_{VS} \Psi_j\, R\, dV \right] \Psi_j = 0,$$

which requires

$$b_k = (B_k^2 - B^2)^{-1} \int_{VS} \Psi_k\, R\, dV.$$

Consequently,

$$n(\mathbf{r}) = \sum_{j=1}^{\infty} (B_j^2 - B^2)^{-1}\, \Psi_j \int_{VS} \Psi_j\, R\, dV \qquad (6)$$

is a representation of the desired solution for Eq. (1).

3. CRITICALITY OF REACTORS

The mathematical properties of the solution (6) will determine what values of B^2 are physically acceptable for the operation of a reactor in a steady state. For a simple one-dimensional case illustrating Eq. (6), see Appendix I.

Case 1: $0 < B^2 < B_1^2 < B_2^2 < \cdots$; The Subcritical Reactor

In this case, we have taken the material buckling B^2, as determined by the material of the reactor, to be *less* than the geometrical buckling, B_1^2, as determined by the shape of the reactor, so that the material buckling is less than any of the mathematically permissible constants B_k^2 of the sequence $\{B_k^2\}$. This means that each term of the infinite series

$$\sum_{j=1}^{\infty} \Psi_j(\mathbf{r}) \int_{VS} \Psi_j(\mathbf{r})\, R(\mathbf{r})\, dV \qquad (7)$$

is divided by a positive constant, $B_k^2 - B^2$, and these new constants form an infinite sequence $\{B_k^2 - B^2\}$. Now the infinite series in Eq. (7) is just the development of the source function $R(\mathbf{r})$ in the Fourier manner:

$$R = \sum_{j=1}^{\infty} \Psi_j \int_{VS} \Psi_j\, R\, dV, \qquad (8)$$

which is either uniformly or conditionally convergent to a nonnegative function. For the series (6), each term of the series in Eq. (8) has been divided by an ever increasing constant so that the convergence should be improved in Eq. (6) over that in Eq. (8) to a *nonnegative definite function*. (This can be proved rigorously, and a proof is given in Appendix IV.) We have, therefore, the important result: When a finite, homogeneous, bare reactor with a material buckling which is positive and less than the geometrical buckling contains a distributed nonnegative finite source of neutrons, the final steady state of the neutron density in the reactor is finite everywhere in the reactor.

We obtain two further conclusions from this analysis. Suppose $(B_1^2 - B^2) \to 0$, then the first term of Eq. (6), which is necessarily positive, becomes indefinitely large, in fact, as large as one pleases as $B_1^2 - B^2$ is made as small as one pleases. The question arises whether the steady-state density also increases indefinitely. We prove the answer is affirmative, for consider the integral

$$\int_{VS} n\, \Psi_1 dV = (B_1^2 - B^2)^{-1} \int_{VS} \Psi_1\, R\, dV, \qquad (9)$$

which follows from the orthonormality relations

$$\int_{VS} \Psi_m\, \Psi_n\, dV = \delta_{mn} \qquad (10)$$

among the functions of the sequence $\{\Psi_k\}$. (Another derivation of Eq. (9) appears in Appendix II.) The result expressed by Eq. (9) is

the analytical formulation of the weighted steady-state density integrated over the reactor. The weighting function is the fixed fundamental solution Ψ_1, nonnegative definite. Hence, the result of the integration is that, since

$$(B_1^2 - B^2)^{-1} \int_{VS} \Psi_1 R dV$$

can become as large as one pleases by making B^2 sufficiently close to B_1^2, the magnitude of n must increase as large as one pleases and, being a necessarily continuous function, it will increase everywhere except on R as long as R is nonnegative definite. *Thus, any large allowable neutron density level in a subcritical reactor can be attained by use of a source* if the material buckling is properly chosen.

Next, suppose $B^2 = 0$. Then the solution for the problem

$$\left.\begin{aligned} &\nabla^2 n(\mathbf{r}) + R(\mathbf{r}) = 0, \\ &n(\mathbf{r}_S) = 0, \quad n(\mathbf{r}) \geqslant 0, \quad R(\mathbf{r}) \geqslant 0 \end{aligned}\right\} \qquad (1')$$

is

$$n = \sum_{j=1}^{\infty} B_j^{-2} \Psi_j \int_{VS} \Psi_j R \, dV \qquad (6')$$

and gives the neutron density distribution in the *equivalent of a pure diffuser* of neutrons which has a perfectly absorbing boundary. As this has no illustration in practice, it is only of academic interest and is discussed further in Appendix III.

A source of neutrons other than fissionable material is seen to be of immediate practical utility in approaching a critical state for relatively small enriched reactors because it enables the operator to determine quickly and conveniently how close the reactor may be to criticality without the danger associated with exceeding it. We mention, in passing, the well-known but rarely explicitly stated fact that a *subcritical reactor cannot be made critical by a nonfissionable source of neutrons no matter how dense that source may be.*

The foregoing analysis indicates that any practical neutron density level may be achieved in a subcritical reactor by variation of B^2, B_1^2, of R. The material buckling B^2 is most easily varied (although not homogeneously) by means of control rods of high neutron absorption cross section. Truly homogeneous fluid reactors may conceivably have the material buckling varied by change in concentration of the fissionable isotope. Geometrical buckling B_1^2 is controlled readily in a fluid or slurry reactor but this is not feasible for fixed reactors of thermal type. However, in reactors called fast or intermediate, which operate primarily on neutrons of epithermal energies for which there are no effective absorbers other than fissionable materials, control is mandatory by variation of B_1^2. This is accomplished by bringing subcritical portions of the reactor closer or farther apart. Finally, the variation of R itself is the least practical but its location in the reactor is quite important in governing the effectiveness of neutron supply rate.

We may summarize the discussion so far in a concise manner. Let us suppose that on the whole a reactor is characterized by generating k neutrons net following the introduction of one neutron counting all possible phenomena. The consequence will be that eventually there will have been generated

$$k + k^2 + \cdots + k^n + \cdots = K$$

neutrons. This sum is convergent if $k < 1$, and

$$K = (1-k)^{-1} k, \quad k < 1, \qquad (11a)$$

is called the total multiplication of the reactor. Hence, when a reactor is subcritical, $k < 1$, the introduction of a source producing S neutrons/second will eventually result in the reactor producing in the steady state at the rate

$$A \sim (K \times R) \text{ neutrons/second} = \text{activity}, \qquad (11b)$$

which will have any allowable value by proper choice of k. This gives the important result again that a nonfissionable source of neutrons permits the achievement of a high density level without having to reach criticality, $k = 1$.

The practical procedure in assembling a reactor to criticality is to plot the reciprocal activity of the steady state *versus* mass of fissionable isotope. When the reciprocal activity $1/A$ is sufficiently close to zero, the operator can estimate the critical mass by extrapolation. This will guide the final assembly.

Case 2: $B^2=B_1^2$ or $k=1$; The Critical Reactor

It is evident from Eqs. (6) or (9) that $n\to\infty$ as $B^2\to B_1^2$ because Ψ_1 and R are nonnegative definite. We conclude that a *critical reactor with a source* ultimately results in catastrophe. The practical consequence is that when a reactor is brought to and kept at criticality, the source must be removed so that the reactor may be operated self-sustaining at or near the level attained. As it is impossible to prevent the supply of neutrons by spontaneous fission, cosmic-ray secondaries, etc., it is evident that, strictly considered, a going reactor operates actually subcritical on this account alone.

We have seen that when the reactor is ideally critical, then density distribution is given by

$$n(\mathbf{r})=c\,\Psi(B_1^2,\mathbf{r}),\ \text{(reactor critical)},$$

where c is an arbitrary nonnegative constant, whereas when the reactor is subcritical,

$$n(\mathbf{r})=\sum_{j=1}^{\infty}(B_j^2-B^2)\Psi_j\int_{VS}\Psi_j\,R\,dV,\qquad\text{(reactor subcritical)}.$$

The distinction between these two cases primarily concerns the following:

(a) safety,
(b) neutron distribution at activity or neutron level desired,
(c) control.

Other factors being equal, it is preferable to operate the reactor subcritical for safety reasons—in general the subcritical reactor cannot of itself increase in power level to dangerous values. However, the fluctuations in k because of temperature, barometric pressure, etc., are significant if k differs only slightly from unity.

When it is necessary to construct a reactor which possesses large voids, gaps, or other structural inhomogeneities, the critical neutron distribution can be only approximately predicted. To make neutron density distribution measurements by activation of small foils at moderate power, it is necessary to operate the reactor without artificial sources within or near it so that local distortions may be avoided. If the reactor is only slightly under the critical condition, $B^2+\epsilon=B_1^2$, then the first term of the expansion in Eq. (6) would be the completely dominant contribution to the distribution. For this reason, at high power levels, the neutron distribution is virtually identical with the fundamental distribution Ψ_1, even with the presence of a source which thus need not be removed.

As a final remark in this case, we note that the analytic result $n\to\infty$ as $B^2\to B_1^2$ is consistent with the expression in Eq. (11):

$$A\sim(1-k)^{-1}k\,R\to\infty\quad\text{as}\quad k\to1.$$

Also, if $R\to0$ and $k\to1$, then A is indeterminate which is another way of expressing the arbitrary nature of the constant c in Eq. (4).

We need not consider the possibility of an indeterminacy arising from

$$\int_{VS}\Psi_1\,R\,dV=0,\quad B_1^2=B^2,$$

in the first term of Eq. (6) because the function R would have to assume negative values in a finite region in order to be orthogonal to an everywhere nonnegative function, and an R with negative values is physically excluded.

Case 3: $B^2>B_1^2$ or $k>1$; The Supercritical Reactor

For purposes of mathematical completeness, we distinguish between the cases where $B_1^2<B^2\neq B_k^2$, $k=2,\ 3,\ 4,\ \cdots$ and $B_1^2<B^2=B_k^2$, $k=2,\ 3,\ 4,\ \cdots$.

(a) $B_1^2<B^2\neq B_k^2$, $k=2,\ 3,\ 4,\ \cdots$.

While a mathematical solution exists for Eq. (1) in this case, it is physically meaningless because as we will show directly, the solution, while finite everywhere, must take on negative values over a finite region not including the boundary R. To prove this consider again the relation given above in Eq. (9):

$$\int_{VS}\Psi_1\,n\,dV=(B_1^2-B^2)^{-1}\int_{VS}\Psi_1\,R\,dV.$$

When $B^2>B_1^2$, the right-hand side of Eq. (9) is negative definite because Ψ_1 and R are nonnegative definite. In the left-hand side, the factor n must be negative over a finite region in order to have a negative integral when weighted

by a non-negative function Ψ_1. As this is physically impossible, it follows that a steady-state balance equation such as Eq. (1) does not exist for a supercritical reactor. In other words, a *supercritical reactor can be described properly only as a time-dependent system.*

(b) $B_1^2 < B^2 = B_k^2$, $k = 2, 3,$ or $4 \cdots$.

To handle this case, we must turn to the general solution (6):

$$n(\mathbf{r}) = \sum_{j=1}^{\infty} (B_j^2 - B^2)^{-1} \Psi_j(\mathbf{r}) \int_{VS} \Psi_j(\mathbf{r}) R(\mathbf{r}) \, dV.$$

Once again we find the solution is unbounded if R is finite, piecewise continuous provided, however, that

$$\int_{VS} \Psi(B_k^2, \mathbf{r}) R(\mathbf{r}) \, dV \neq 0$$

for otherwise, the kth term would become an indefinite form, 0/0, whose limit could only be given for actual cases. Nevertheless, should the right-hand side of Eq. (6) still have a finite result for the possibility that

$$\int_{VS} \Psi(B_k^2, \mathbf{r}) R(\mathbf{r}) \, dV = 0, \quad B_k^2 = B^2,$$

the result would still be physically unacceptable by the same argument advanced above to show n would be required to be negative in a finite region. Therefore, from every viewpoint we find $B^2 > B_1^2$ leads to no possibility of a steady state.

The generalization of these results to multigroup theory and to the consideration of the transient state in a reactor with nonfissionable sources is beyond the original objective of this discussion. For an extended treatment involving the ideas discussed in this paper, the reader is referred to an article by Weinberg and Schweinler.[1]

APPENDIX I

A simple illustration in concrete form of the result that the solution of Eq. (1) is positive definite in the interior and vanishes on the boundary is given here. Assume a slab reactor so that its thickness is very small compared with the length and breadth. Let the opposite large faces normal to the thickness be located at $x = 0$ and $x = L$. We have here virtually one-dimensional geometry and Eq. (1) can be written

$$\left.\begin{aligned} n''(x) + B^2 n(x) + R = 0, \quad B^2 > 0; \\ R = \text{constant} > 0, \\ n(0) = n(L) = 0. \end{aligned}\right\} \quad \text{(I, 1)}$$

The solution is readily verified to be

$$n(x) = (R/B^2)[(\cos B(x - \tfrac{1}{2}L)/\cos \tfrac{1}{2}BL) - 1]. \quad \text{(I, 2)}$$

When the slab is critical, the sequence of permissible solutions is

$$\{\Psi(B_k^2, x)\} = \{(2/L)^{\frac{1}{2}} \sin(k\pi x/L)\}$$

and the sequence of permissible values is

$$\{B_k^2\} = \{(k\pi/L)^2\}; \quad k = 1, 2, 3, \cdots.$$

Hence, the fundamental solution is

$$\Psi_1(x) = (2/L)^{\frac{1}{2}} \sin(\pi x/L) \quad \text{(I, 3)}$$

and the geometrical buckling is

$$B_1^2 = \pi^2/L^2. \quad \text{(I, 4)}$$

For the subcritical slab, $B < \pi/L$. Let us assume

$$B = \epsilon\pi/L, \quad 0 < \epsilon < 1.$$

Substituting this into Eq. (I, 2), we are led to the expression

$$n(x) = (R/B^2)\left(\frac{\cos \epsilon\pi(2x - L)/2L}{\cos \frac{1}{2}\epsilon\pi} - 1\right), \quad \text{(I, 5)}$$

where it is seen on inspection that, for $0 \leqslant x \leqslant L$, n vanishes at $x = 0, L$, satisfies Eq. (I, 1) and is positive definite for $0 < x < L$. The formal equivalence of Eq. (I, 2) and the series solution given by Eq. (6) in the text is the relation

$$\frac{\cos B(x - \frac{1}{2}L)}{\cos \frac{1}{2}BL} = 1 + \frac{4}{\pi} \sum_{j=1}^{\infty} \frac{\sin(2j-1)\pi x/L}{(2j-1)\left[\dfrac{(2j-1)^2\pi^2}{B^2L^2} - 1\right]}. \quad \text{(I, 6)}$$

[1] A. M. Weinberg and H. C. Schweinler, Phys. Rev. **74**, 851 (1948).

APPENDIX II

Another derivation of the relation Eq. (9)

$$\int_{V_S} n(\mathbf{r})\,\Psi(B_1^2,\mathbf{r})\,dV = (B_1^2-B^2)^{-1}\int_{V_S}\Psi(B_1^2,\mathbf{r})\,R(\mathbf{r})\,dV$$

is given here by means of Green's theorem in 3 dimensions. We start with Eqs. (1):

$$\nabla^2 n+B^2 n+R=0,$$
$$n(\mathbf{r}_S)=0,$$

and the fundamental solution from Eq. (3):

$$\nabla^2\Psi_1+B_1^2\Psi_1=0,$$
$$\Psi_1(\mathbf{r}_S)=0.$$

We form

$$\Psi_1\,\nabla^2 n+B^2\Psi_1\,n+\Psi_1 R=0,$$
$$n\nabla^2\Psi_1+B_1^2 n\,\Psi=0,$$

and subtract and integrate over the volume of the reactor, so that

$$\int_{V_S}(n\,\nabla^2\Psi_1-\Psi_1\,\nabla^2 n)\,dV + (B_1^2-B^2)\int_{V_S} n\,\Psi_1\,dV=\int_{V_S}\Psi_1\,R\,dV.$$

By Green's theorem

$$\int_{V_S}(n\nabla^2\Psi_1-\Psi_1\nabla^2 n)\,dV = \int_S(n\nabla\Psi_1-\Psi_1\nabla n)\cdot d\mathbf{S}=0,$$

because n and Ψ_1 both vanish on the boundary R. This gives again Eq. (9):

$$\int_{V_S} n\,\Psi_1\,dV=(B_1^2-B^2)^{-1}\int_{V_S}\Psi_1\,R\,dV,$$

so that no use was made of the actual solution for n given by Eq. (6).

A concluding remark in this connection is to point out that relation (9) cannot be used to establish the nonnegative character of n, when $0<B^2<B_1^2$ because from Eq. (9), when the right-hand side is positive, it does not necessarily follow that n must be nonnegative although it does follow that n must be negative over a finite region when the right-hand side is negative. Because Eq. (9) was not sufficient to establish the nonnegative character of n for nonnegative R, we were forced to consider the representation in Eq. (6) in order to establish this obvious physical fact in a rigorous mathematical way.

APPENDIX III

For the sake of completeness we record the representation of the neutron density distribution in a bare reactor when the source is supposed to be contracted to a point but of such "infinite" density as to be a finite supply rate of neutrons. The simplest thing to do is to assume a supply rate of 1 neutron per second. How can such a source density be represented mathematically? From the point of view of the physicist such a function is extremely convenient, although not essential to the mathematician. The point-source has been introduced into physical literature by several physicists. According to P. A. M. Dirac, it is called the delta-function, and is *defined* to have the properties:

(a) $\delta(\mathbf{r},\mathbf{r}')=0, \quad \mathbf{r}\neq\mathbf{r}';$

(b) $$\int_{V_S}\delta(\mathbf{r},\mathbf{r}')dV=\begin{cases}1 \text{ if } V_S \text{ includes } \mathbf{r}',\\ 0 \text{ if } V_S \text{ does not include } \mathbf{r}';\end{cases}$$

(c) $$\int_{V_S}\delta(\mathbf{r},\mathbf{r}')\,R(\mathbf{r})dV=\begin{cases}R(\mathbf{r}'), \text{ if } V_S \text{ includes } \mathbf{r}',\\ 0, \text{ if } V_S \text{ does not include } \mathbf{r}'.\end{cases}$$

A. Sommerfeld has called the point-source the "zackenfunction" and, when applied to vibration theory as a force-function, it has been called a point-load by A. G. Webster.

We are led to consider the following problem:

$$\nabla^2\Gamma(\mathbf{r},\mathbf{r}')+B^2\Gamma(\mathbf{r},\mathbf{r}')+\delta(\mathbf{r},\mathbf{r}')=0,$$
$$\Gamma(\mathbf{r}_S,\mathbf{r}')=0, \quad 0<B^2<B_1^2, \qquad \text{(III, 1)}$$

where $\Gamma(\mathbf{r},\mathbf{r}')$ designates the density distribution for the δ-function source at $\mathbf{r}'$.

To obtain a solution, assume a series solution

$$\Gamma(\mathbf{r},\mathbf{r}')=\sum_{j=1}^{\infty} C_j\,\Psi(B_j^2,\mathbf{r})$$

in terms of the complete orthonormal sequence $\{\Psi(B_k^2, \mathbf{r})\}$. This leads to the equation

$$\sum_{j=1}^{\infty}(B^2-B_j^2)C_j\Psi_j+\delta(\mathbf{r}, \mathbf{r}')=0. \quad \text{(III, 2)}$$

Multiplication of Eq. (III, 2) by Ψ_k and integration over the reactor volume V_S leads to $C_k=(B_k^2-B^2)^{-1}\Psi(B_k^2, \mathbf{r})$ and

$$\Gamma(\mathbf{r}, \mathbf{r}') = \sum_{j=1}^{\infty}(B_j^2-B^2)^{-1}\Psi(B_j^2, \mathbf{r})\Psi(B_j^2, \mathbf{r}'). \quad \text{(III, 3)}$$

For the problem $(B^2=0)$,

$$\left.\begin{aligned}\nabla^2 G(\mathbf{r}, \mathbf{r}')+\delta(\mathbf{r}, \mathbf{r}')=0,\\ G(\mathbf{r}_S, \mathbf{r}')=0,\end{aligned}\right\} \quad \text{(III, 4)}$$

G would be the distribution for pure diffusion, and in a manner similar to the above, or directly from Eq. (III, 3), we get

$$G(\mathbf{r}, \mathbf{r}')=\sum_{j=1}^{\infty}B_j^{-2}\Psi(B_j^2, \mathbf{r})\Psi(B_j^2, \mathbf{r}'). \quad \text{(III, 5)}$$

The representations (III, 3) and (III, 5) are known as Green's functions or influence functions. The latter designation is expressive of the neutron density arising in the steady state at point $\mathbf{r}$ due to a unit source at $\mathbf{r}'$ and vice versa. Hence, if the source in the reactor is continuously distributed with source density $R(\mathbf{r})$ throughout the reactor, the solutions (6) and (6′) represent precisely the integrated effect of the distributed source density function $R(\mathbf{r})$. For this reason the solutions (6) and (6′) are said to be represented *source-wise*. Notice that both $G(\mathbf{r}, \mathbf{r}')$ and $\Gamma(\mathbf{r}, \mathbf{r}')$ are completely symmetric in $\mathbf{r}, \mathbf{r}'$.

The formal representation of $\delta(\mathbf{r}, \mathbf{r}')$ as a series is given by

$$\delta(\mathbf{r}, \mathbf{r}')=\sum_{j=1}^{\infty}\Psi(B_j^2, \mathbf{r})\Psi(B_j^2, \mathbf{r}'). \quad \text{(III, 6)}$$

APPENDIX IV

We start with the problem of pure diffusion from a unit point source:

$$\left.\begin{aligned}\nabla^2 G(\mathbf{r}, \mathbf{r}')+\delta(\mathbf{r}, \mathbf{r}')=0,\\ G(\mathbf{r}_S, \mathbf{r}')=0.\end{aligned}\right\} \quad \text{(IV, 1)}$$

The solution,

$$G(\mathbf{r}, \mathbf{r}')=\sum_{j=1}^{\infty}\lambda_j^{-1}\Psi(\lambda_j, \mathbf{r})\Psi(\lambda_j, \mathbf{r}') \quad \text{(IV, 2)}$$

(where we have written $\lambda_k=B_k^2$) from Eq. (III, 5), can also be written in the form[2]

$$G(\mathbf{r}, \mathbf{r}')=(4\pi|\mathbf{r}-\mathbf{r}'|)^{-1}+\mathcal{G}(\mathbf{r}), \quad \text{(IV, 3)}$$

where $\mathcal{G}(\mathbf{r})$ is a function harmonic everywhere within and on the boundary R and on the boundary has the value

$$\mathcal{G}(\mathbf{r}_S)=-(4\pi|\mathbf{r}-\mathbf{r}'|)^{-1}.$$

If the δ-function source at $\mathbf{r}'$ is surrounded by an infinitesimal sphere σ, then in the closed region bounded by σ and R, the function $G(\mathbf{r}, \mathbf{r}')$ must attain its maximum value on σ and minimum value on R because it is harmonic in the region (the singularity at $\mathbf{r}'$ has been excluded by σ). As a consequence, $G(\mathbf{r}, \mathbf{r}')>0$ within R and goes to infinity at $\mathbf{r}'$ as $(4\pi|\mathbf{r}-\mathbf{r}'|)^{-1}$.

Now we consider the special case of the multiplying medium (subcritical reactor) with the unit point source (Appendix III):

$$\nabla^2\Gamma(\mathbf{r}, \mathbf{r}')+\lambda\,\Gamma(\mathbf{r}, \mathbf{r}')+\delta(\mathbf{r}, \mathbf{r}')=0,$$
$$\Gamma(\mathbf{r}_S, \mathbf{r}')=0,$$

for which we have found the expansion

$$\Gamma(\mathbf{r}, \mathbf{r}')=\sum_{j=1}^{\infty}(\lambda_j-\lambda)^{-1}\Psi(\lambda_j, \mathbf{r})\Psi(\lambda_j, \mathbf{r}').$$

Moreover, we must have here too that

$$0<\lambda<\lambda_1<\lambda_2<\cdots$$

to avoid the certainty of negative values for $\Gamma(\mathbf{r}, \mathbf{r}')$ when $\lambda>\lambda_1$, as is evident from the equality

$$\int_{V_S'}\Gamma(\mathbf{r}, \mathbf{r}')\Psi(\lambda_1, \mathbf{r}')\,dV'=(\lambda_1-\lambda)\Psi(\lambda_1, \mathbf{r})$$

as discussed in Appendix II.

[2] A. G. Webster, *Partial Differential Equations of Mathematical Physics* (Teubner, Leipzig, 1933), p. 233.

We prove that $\Gamma(\mathbf{r}, \mathbf{r}') \geqslant 0$ if $0<\lambda<\lambda_1<\cdots$ as follows:

$$(\lambda_k-\lambda)^{-1}=\lambda_k^{-1}\,(1+\lambda\lambda_k^{-1}+\lambda^2\lambda_k^{-2}+\cdots)$$

so that

$$\Gamma(\mathbf{r}, \mathbf{r}')=\sum_{j=1}^{\infty}\lambda_j^{-1}\,\Psi(\lambda_j, \mathbf{r})\,\Psi(\lambda_j, \mathbf{r}')$$

$$+\lambda\sum_{j=1}^{\infty}\lambda_j^{-2}\,\Psi(\lambda_j, \mathbf{r})\,\Psi(\lambda_j, \mathbf{r}')$$

$$+\lambda^2\sum_{j=1}^{\infty}\lambda_j^{-3}\,\Psi(\lambda_j, \mathbf{r})\,\Psi(\lambda_j, \mathbf{r}')+\cdots,$$

which we write by rearrangement as

$$\Gamma(\mathbf{r}, \mathbf{r}')=G_1(\mathbf{r}, \mathbf{r}')+\lambda\,G_2(\mathbf{r}, \mathbf{r}')+\lambda^2\,G_3(\mathbf{r}, \mathbf{r}')+\cdots \quad \text{(IV, 4)}$$

with

$$G_1(\mathbf{r}, \mathbf{r}')\equiv G(\mathbf{r}, \mathbf{r}'),$$

$$G_2(\mathbf{r}, \mathbf{r}')=\int_{V_S''} G_1(\mathbf{r}, \mathbf{r}'')\,G_1(\mathbf{r}', \mathbf{r}'')\,dV'', \quad \text{(IV, 5)}$$

$$\cdots\cdots$$

$$G_k(\mathbf{r}, \mathbf{r}')=\int_{V_S''} G_1(\mathbf{r}, \mathbf{r}'')\,G_{k-1}(\mathbf{r}', \mathbf{r}'')\,dV''$$

$$\cdots\cdots.$$

Consequently $\Gamma(\mathbf{r}, \mathbf{r}') \geqslant 0$, as it is clear from Eq. (IV, 5) and the fact $G_1(\mathbf{r}, \mathbf{r}') \equiv G(\mathbf{r}, \mathbf{r}') \geqslant 0$ throughout V_S, that $\Gamma(\mathbf{r}, \mathbf{r}')$ is the sum of everywhere nonnegative terms.

At this stage, it is simple to see that $n(\mathbf{r}) \geqslant 0$, where

$$n(\mathbf{r})=\int_{V_S''}\Gamma(\mathbf{r}, \mathbf{r}')\,R(\mathbf{r}')\,dV' \quad \text{(IV, 6)}$$

because we have shown $\Gamma(\mathbf{r}, \mathbf{r}')$ and $R(\mathbf{r}')$ are everywhere nonnegative.

Echo Ranging with Audiofrequencies

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(Received July 14, 1952)

The mathematical form of a propagating wave is discussed, and it is pointed out that it may be misleading to emphasize the harmonic form of a wave in the general physics course. A demonstration piece utilizing a sound pulse is described. The demonstration piece is suitable to illustrate (a) the speed of a sound, (b) reflection methods of ranging, and (c) reverberations in rooms.

ALMOST invariably the writer of the elementary physics textbook chooses to discuss sound waves in air as periodic disturbances propagated by compressional plane waves of sinusoidal form. He demonstrates the equation for such a wave to be $A(x, t)=A_0\sin(2\pi/\lambda)\times(x-vt)$ and immediately deduces the well-known formula relating velocity of propagation, wavelength, and frequency. This is by no means incorrect, but in a certain sense it may be misleading for it implies that all sounds have a frequency, and almost suggests a necessary relationship between wave motion and periodicity.

This is of course not true. The well-known equation which describes propagating waves in a nondissipative medium given as

$$\frac{1}{V^2}\frac{\partial^2 D(\bar{r}, t)}{\partial t^2}=\nabla D^2(\bar{r}, t)$$

reduces in the case of plane waves to

$$\frac{1}{V^2}\frac{\partial^2}{\partial t^2}A(x, t)=\frac{\partial^2}{\partial x^2}A(x, t).$$

It is clear that the disturbance $A(x, t)$, whatever its nature might be, is, in general, described by a function

$$A(x, t)=f_1(x-vt)+f_2(x+vt).$$

It is apparent that any functions f_1 and f_2 will

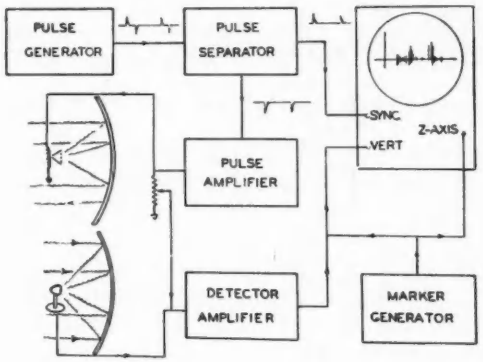

FIG. 1. Block diagram of echo ranging apparatus.

satisfy the equation as long as their appropriate derivatives exist and their arguments contain space and time as $(x \pm vt)$. The sine wave function is of course only one special example of such a function and approximations to it are easily studied in the acoustical laboratory. In fact, all of the classical sound experiments in the general physics laboratory and most of the classroom demonstrations with which the writer is familiar employ these nearly periodic waves. This unfortunate concentration on a particular kind of wave may tend to cultivate the illusion in the mind of the beginning student that wave motion and periodicity are necessary partners.

The object of this paper is therefore to describe a rather simply built demonstration piece which avoids the above-mentioned difficulty by utilizing a sound pulse. Essentially the apparatus contains the same elements as ultrasonic echo ranging device used by submarine search patrols. A sound pulse is produced, and as it is propagated from the source it may encounter obstacles. Reflections occur and return a portion of the

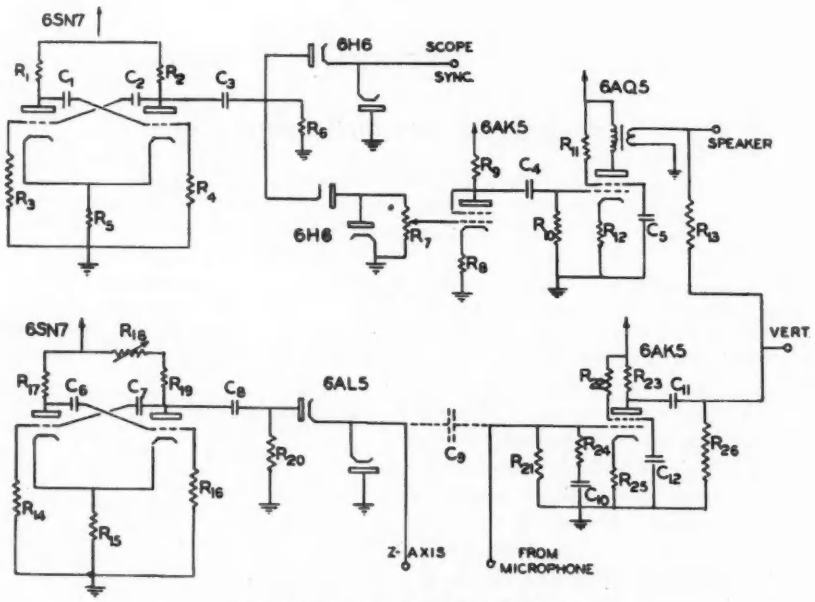

FIG. 2. Schematic diagram of apparatus.

R_1, R_2, R_{17}, R_{19}; 15 000 ohms, 1 watt.
R_3, R_4, R_{13}, R_{14}, R_{16}; 100 000 ohms.
R_5, R_8, R_{15}, R_{23}; 330 ohms.
R_6, R_{10}, R_{20}, R_{21}, R_{26}; 50 000 ohms.
R_{12}; 50 ohms. R_{18}, variable 10 000 ohms.
R_9, R_{22}; 30 000 ohms. R_{11}, 12 000 ohms. R_{24}, 2000 ohms.
R_7; 500 000-ohm potentiometer.
C_1, C_6, C_{12}; 0.01 μf. C_2, 2 μf. C_3, 0.001 μf. C_4, 0.0002 μf.
C_5, 0.1 μf. C_8, 0.01 μf. C_7, C_8, C_{10}, 0.0005 μf.
C_9, Stray capacitance.

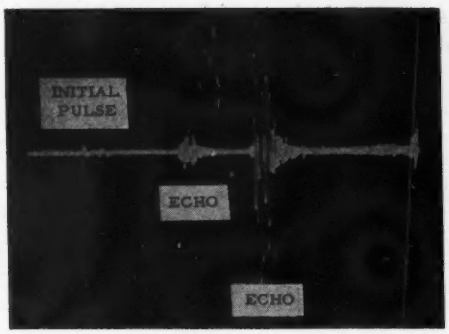

FIG. 3. Photograph of single oscilloscope trace showing pulses corresponding to the initial pulse and subsequent pulses due to reflections.

FIG. 4. Photograph similar to Fig. 3, but with electronically produced range markers indicating each meter of range. First obstacle is approximately 2.2 meters distant and the second approximately 4.1 meters.

pulse to the detector. When the echo is presented in time relation to the original pulse, the delay between the initial pulse and the echo is a measure of the distance to the reflecting obstacle.

In order that the whole sequence of events may be seen on an oscilloscope trace, a suitable circuit trips the sweep of the oscilloscope several milliseconds before the sound source is pulsed. A separate electronic time scale may be placed on the scope face by means of the marker switch. The markers are spaced at 5.8 milliseconds in time and each represents one meter of range. The transmitter is a two-inch permanent magnet speaker and the detector is a dynamic microphone. Directivity of the transmitter and the detector is obtained by means of suitable paraboloidal reflectors. Thus the azimuth as well as the range of the reflecting obstacle may be determined.

Figure 1 shows the functional block diagram and Fig. 2 the schematic diagram. Figures 3 and 4 are pictures of the range presentation without and with markers, respectively.

The apparatus can be used to demonstrate several things. (a) By knowing the length of path of the sound wave, the speed of sound may be calculated from the time. It is convenient and more precise to use a precision sine wave oscillator to synchronize the marker generator. (b) By using a nondirectional transmitter and microphone, it may be used to measure the reverberation pattern and reverberation time in a room. (c) It may be used to illustrate the principles of ultrasonic submarine echo ranging and radar.

Aside from the oscilloscope (any scope with external synchronization input and Z axis input for brightening is suitable) the circuit consists of eight tubes and its versatility will repay the few hours that it takes to build. The voltage pulses are produced by differentiating the square wave output of an asymmetrical multivibrator (6SN7). This output consists of a short positive pulse followed in several milliseconds by a short negative pulse. Two duo-diodes separate the positive and negative pulses, the positive one becoming the sweep trigger pulse and the second amplified to drive the transmitter. The pentode-connected 6AK5 is a voltage amplifier bringing up the microphone voltage to a level where it may actuate the scope deflection. Another asymmetrical multivibrator generates the marker pulses. The markers are applied to the Y axis amplifier on top of the echo signal from the detector. The markers are brightened and made more visible by applying a portion of the marker voltage to the Z axis input of the oscilloscope. Since, when the speed of sound is approximately 340 meters/second at room temperature, the markers are applied at a frequency of 170 cycles/second, a simple method of calibrating the marker distance is to tune the frequency of the multivibrator (by adjusting R_{18}) till it is in tune with the 170-cycle note produced by a fork or frequency generator.

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics

52. Frontispiece and Vignette from Chérubin's *La dioptrique oculaire*

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(Received June 2, 1952)

Chérubin's *La dioptrique oculaire* (Paris 1671) is an unusually beautiful scientific work. The frontispiece and a vignette engraved by Edelinck are here reproduced and commented upon.

LA dioptrique oculaire by Père CHÉRUBIN d'Orléans is one of the more important 17th-century works on optics. The author showed great ingenuity and skill in the design and construction of optical instruments and he advanced the optics of his day considerably through the instrumental improvements for which he was responsible. He perfected the binocular telescope and invented the binocular microscope as well as the rotating objective carrier. His handsome folio work, published at Paris in 1671, should be especially useful to those who wish to become familiar with 17th-century methods of grinding glass and constructing telescopes and microscopes.

FIG. 1. Frontispiece from Chérubin's *La dioptrique oculaire*.

Almost nothing is known about CHÉRUBIN's life except that he was a Capuchin monk of a convent at Orléans. Even the dates of his birth and death are unknown. He did, however, publish several other works in addition to *La dioptrique oculaire*, one of which entitled *La vision parfaite* went through several editions (Paris, 1677, 1678, 1681). While he applied himself principally to optics, he also gave attention to the science of acoustics and invented an instrument by which he was able to assist the hearing to a great degree as well as some sort of a telegraphic instrument. One of his published works, moreover, deals with experiments on the vacuum and the pressure of air.

La dioptrique oculaire is noteworthy not only as one of the best of the 17th-century publications on optics but also because it is one of the very few books illustrated by GÉRARD EDELINCK (1640–1707), the great portrait engraver, whose work has in many respects never been excelled. EDELINCK engraved in all over 300 plates, more than 200 of which were portraits, but he illustrated only about half a dozen books.

The splendid allegorical frontispiece of *La dioptrique oculaire* (reproduced as Fig. 1) was drawn by JEAN LE PAUTRE (1618–1682) and beautifully engraved by EDELINCK. It depicts

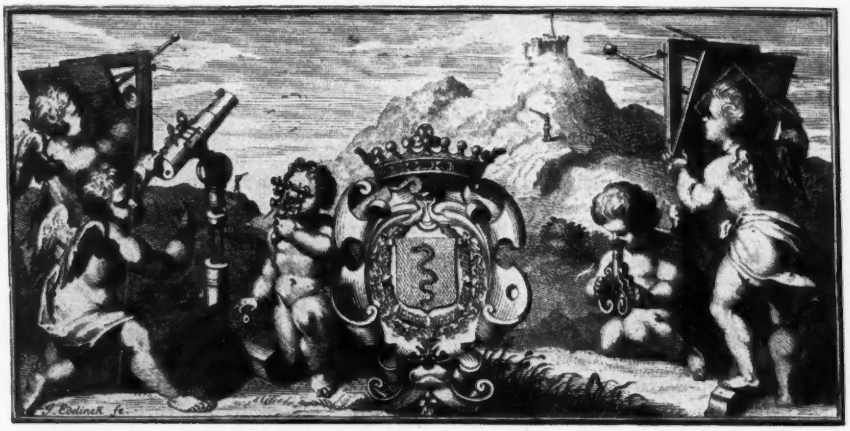

FIG. 2. Vignette from Chérubin's *La dioptrique oculaire.*

three angels descending from heaven to present telescopes and microscopes to mortal man with the admonition, *Videte opera domini.* Six walks of life are clearly represented by the six figures in the foreground; on the right the natural philosopher with his astrolabe, the physician with his anatomical drawing, and the artist with his palette and brushes; on the left the soldier with an ancient sighting tube (*tuyau oculaire*), the engineer with a plan of a fortification, and a sailor with a cross staff.

EDELINCK also engraved the charming vignette that heads the dedication to COLBERT (here reproduced as Fig. 2). It shows cherubs using perspectographs, a binocular telescope, a small binocular, and a microscope. Perspectographs were very popular during the latter half of the 17th century; SIR CHRISTOPHER WREN greatly improved their design and PEPYS records in his diary (April 30, 1669) that he was having one made. The telescope, binocular, and microscope depicted are those designed by CHÉRUBIN and described in *La dioptrique oculaire.* The binocular shows the mechanism he introduced for adjusting to the interval between the eyes of the user. The microscope is supported on three scroll supports attached to the central part of the body, which is as a result fixed. The eyepiece slides on this central part at the upper end and the objective does the same at the lower end. By adjusting these two sliding parts the microscope could be focused and the magnification could be varied. Also the tube carrying the objective could be replaced by other tubes with objectives of different powers. In one form of the instrument a screw was provided for focusing.

"Franklin's minute-book, the journal in which he noted daily his ideas and the results of his experiments, has been lost. The only material from which to follow his work and that of his friends and assistants in the remarkable ten years from 1746 to 1756 is contained in various editions, prepared by Franklin himself, of his letters to his friend Peter Collinson of London (who communicated them to the Royal Society) and to other correspondents. These can be described as lying halfway between progress-reports and scientific papers. They are not easy to read; the many new conceptions of electrical phenomena which occupied the mind of the author jostle each other for space and it is sometimes hard to follow their development. At times one is at a loss to understand how he came to suggest some important new idea, until careful examination of a previous letter shows one sentence which provides the link in the chain of his argument, and proves that he was not working on a 'hunch' but on a closely-reasoned placing of experiment upon experiment and argument upon argument."—B. F. J. SCHONLAND, Journal of the Franklin Institute, **253**, 376 (1952).

The Operating Principles of Synchrotron Accelerators

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(Received April 28, 1952)

The synchrotron accelerator has many varieties. Some confusion exists in the widely scattered literature on particle accelerators as to the basic similarities and differences of the various types. This confusion is partly due to indistinct designation. In this review paper each variety is designated and described as a logical derivation from a previous type, and the basic principle of operation of each is described in turn. Factors governing oscillations in phase and in orbit are listed and described. Basic references are presented covering each point.

THE name "synchrotron" suggested by McMillan[1] has been used to cover a number of different devices which are only somewhat similar in operation. This blanket use of the name "synchrotron" is rather unfortunate since it leads to a certain degree of confusion. As suggested by McMillan, it covers any particle accelerator of the synchrotron type in which either the frequency of the accelerating voltage (Type I) or the strength of the guiding magnetic field (Type II) or both (Type III) are varied during the process of energizing each group of particles.

These three types, however, are identical neither in construction nor operation. The writer will attempt to clarify this situation by means of the designations listed below.

Designation in this paper	Designation current in the literature (suggested by Bohm and Foldy, reference 2)
Type I	F-M Cyclotron or synchrocyclotron.
Type II	Synchrotron or electron synchrotron.
Type II-B	Synchrotron with betatron injection.
Type III	Synchrotron or proton synchrotron.

In what follows it will be assumed that the reader is familiar with the two basically different varieties of particle accelerators—the cyclotron and the betatron.[3,4]

CYCLOTRON MODIFICATION

Types I and II were both suggested independently by Veksler[5] and McMillan.[1] As originally proposed, they were essentially modifications of the cyclotron—modifications wherein the frequency of the accelerating voltage (Type I) or the strength of the magnetic field (Type II) were to be varied cyclically, producing pulses of high energy particles. The prime purpose of the modifications was to overcome the effect of relativistic mass increase which, in the cyclotron, introduces a limitation on the energy which can be given to heavy particles and makes it completely impractical for energizing light particles.

The well-known cyclotron equation expressing the equilibrium of forces acting on a freely moving charged particle in a plane perpendicular to a uniform magnetic field H is

$$Hev/c = mr\omega^2, \tag{1}$$

where e is the charge of the particle, c is the velocity of light, v the velocity of the particle, r the radius of its circular path, and ω its angular velocity around the center of the path. The symbol m here denotes the relativistic mass of the particle and is related to its rest mass by the standard equation

$$m = m_0/[1-v^2/c^2]^{\frac{1}{2}}. \tag{2}$$

By introducing the relativistic expression $E = Mc^2$ for the particle's total energy (kinetic+rest) and recognizing that $v = r\omega$ for the assumed path, the basic cyclotron equation may be rewritten in the form

$$\omega = Hce/E. \tag{3}$$

In the conventional cyclotron a radiofrequency electric field is applied across the electrode gaps. If the frequency of the electric field corresponds with that given by Eq. (3), the particle will gain kinetic energy by making successive trips through the gaps. Ultimately, its total energy E will begin to increase appreciably above the

* Now with Northrop Aircraft, Inc., Hawthorne, California.
[1] Edwin M. McMillan, Phys. Rev. **68**, 143 (1945).
[2] David Bohm and L. Foldy, Phys. Rev. **70**, 249 (1946).
[3] E. C. Pollard and W. L. Davidson, *Applied Nuclear Physics* (John Wiley and Sons, Inc., New York, 1942).
[4] J. C. Slater, *Microwave Electronics* (D. Van Nostrand Company, Inc., New York, 1950).
[5] V. Veksler, J. Phys. (U.S.S.R.) **9**, No. 3, 153 (1945).

rest value m_0c^2 causing its rotational frequency to fall out of synchronism with that of the accelerating electric field. It is this feature which limits the energy which can be imparted to particles by the cyclotron. This difficulty is overcome in the various types of synchrotron accelerators.

In the Type I machine H remains constant and the rotational frequency ω is permitted to decrease as the particle gains energy. The frequency of the accelerating field is then decreased in synchronism so that the particle will continue to pass through the gaps during an accelerating portion of the radiofrequency cycle.

In the Type II machine the increase in E is compensated for by a proportional increase in H, thus keeping ω constant. In this way, the particle continues to pass through the gaps during an accelerating portion of the radiofrequency cycle.

Surprisingly, it develops that the exact manner of variation of either the radiofrequency or the magnetic field during the interval of accelerating one group of particles is not at all critical. The requirements are simply slow, monotonic variation and sufficient radiofrequency voltage. These are specified more carefully by Veksler's[5] requirements:

$$\text{For Type I:} \qquad V_0 \gg \frac{Hc}{\omega_n^2}\frac{d\omega_n}{dt},$$

$$\text{For Type II:} \qquad \frac{\omega^2 V_0}{\pi c} \gg \left|\frac{dH}{dt}\right|_{\max},$$

where V_0 is the amplitude of the radiofrequency voltage and ω_n is the frequency of this voltage during the last few revolutions of the particle to be considered.

THE PHASE STABILITY PRINCIPLE

In order to gain some understanding of why the manner of maintaining synchronism between the rotational frequency and the accelerating frequency is not nearly so critical as one might suspect, let us consider the physical picture of the accelerating process for a single particle.

Before any appreciable mass increase occurs the particle travels around with a constant rotational frequency and thus passes through the electrode gaps at the same instant of every rf cycle. Such instants are represented by points A_1 and B_1 in Fig. 1. (According to focusing considerations, these points must be confined to the latter quarter of the positive half-cycle of the accelerating voltage.)

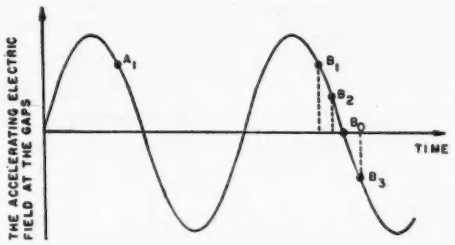

FIG. 1. Sketch showing the phase relationships between the accelerating field and the angular velocities of the particles. A particle which has passed through the gaps initially at instant A_1, may re-enter the next time at B_0, B_1, B_2, or B_3 depending upon its speed of travel around the machine.

If the particle then acquires sufficient energy such that it is traveling at relativistic speeds, its angular velocity will begin to decrease and it may now arrive at the gaps at instant B_2. Here it will continue to gain in energy (though by a smaller amount than before) and decrease in angular velocity until it is finally crossing the gaps at instant B_0. If then it should lag still further behind in phase such that it next crosses the gaps at instant B_3, it will encounter a decelerating voltage and will lose energy to the electric field. This will cause its angular velocity to increase slightly so that the particle is restored to the stable condition of crossing the gaps at instant B_0. The particle then continues to travel around the machine in this condition gaining no further energy.

It can be seen that if the particle is now displaced in its phase (with respect to the phase of the electric field) by less than 90°, its angular velocity will oscillate until it eventually settles down to crossing the gaps again at point B_0. This is the stable condition for a particle traveling at relativistic velocities.[6]

To see how the energy of the particle is increased let us first consider the Type I machine. After the stable condition has been reached, the frequency of the electric field is decreased by a

[6] Actually, the particle in its stable condition executes continuous oscillations around the point of zero phase. This situation is discussed in later paragraphs.

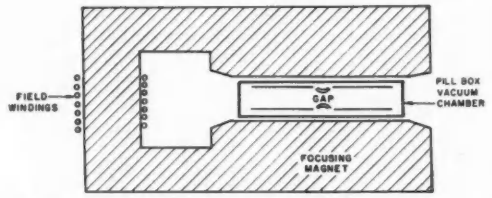

FIG. 2. Cross section of both the Type I and Type II machines showing the "pill-box" vacuum chamber, the focusing magnet, and the gap between the semicircular accelerating electrodes.

small amount. This makes the particle's angular velocity now too high for the new accelerating frequency, and it arrives at the gaps too soon (instant B_2). It will thus gain energy from the accelerating field and will continue to do so until its angular velocity decreases to the point where its rotational frequency again equals the frequency of the electric field. It will then settle down in this condition of increased energy. Thus, if the radiofrequency is slowly but continually decreased, the rotational frequency or angular velocity of the particle will follow as its total energy increases in synchronism.

In the Type II machine the action is quite similar. After the particle has gained sufficient energy from cyclotron action and reaches the stable condition, the magnetic field is increased by a small amount. This raises the angular velocity of the particle and advances it in phase to the point where it crosses the gaps during an accelerating portion of the radiofrequency cycle. This causes the particle to increase in energy. This increase continues until the ratio H/E is restored to its original value at which time the particle has fallen behind in phase until it is once again crossing the gaps at instant B_0. If the magnetic field is increased slowly but continuously, this gain in energy is continuous.

The principle of phase stability was verified experimentally by Richardson and co-workers[7] at Berkeley for a machine of Type I and by Goward and Barnes[8] in England for a machine of Type II.

RADIATION LOSSES

The major limitation upon the peak energy which particles can acquire in the betratron is due to the energy lost from the particles by radiation.[9,10] This presents no limitation in either of the cases described above. If the particle loses energy by radiation in traversing its path once, then its angular velocity will increase slightly and it will become advanced in phase. The particle will then be acted upon by an accelerating field and the energy lost due to radiation will be restored. The stable point under these conditions is slightly ahead of the point of zero phase.

Thus, in machines of Types I or II it is necessary only that the peak value of accelerating voltage exceed the amount necessary to replace the energy lost by radiation per revolution. If this condition is established, the particle will continue to rise in energy in synchronism with either the radiofrequency or the magnetic field.

Actually, the radiation loss has a beneficial effect in these machines by acting as damping for the phase oscillations of the particles.

APPLICATIONS FOR TYPES I AND II

It has been suggested that Type II machine is more suitable for energizing electrons while Type I is best for heavy particles. This is because the large mass change of the electron would necessitate a prohibitive range of frequency variation. On the other hand, the small mass change of the heavy particles (for an equal total energy) permits F-M operation and avoids the very costly laminated iron construction which is essential to the efficient operation of ac electromagnets.

EFFECT OF ORBIT RADIUS ON CONSTRUCTION

No statement has been made thus far with regard to the variation of the particle's orbital

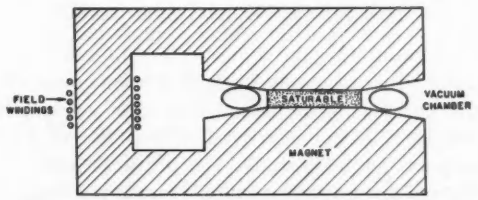

FIG. 3. Cross section of the betatron-synchrotron combination (Type IIA) showing the "doughnut" vacuum chamber, the magnet, and the early-saturating central core.

[7] Richardson, Wright, Lofgren, and Peters, Phys. Rev. **73**, 424 (1948).
[8] Goward and Barnes, Nature **158**, 413 (1946).
[9] Edwin M. McMillan, Phys. Rev. **68**, 145 (1945).
[10] Julian Schwinger, Phys. Rev. **70**, 798 (1946).

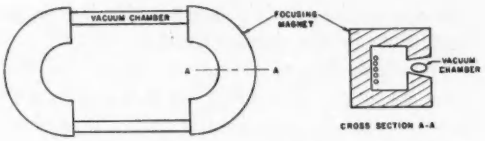

FIG. 4. The Racetrack synchrotron. (Straight-section length is exaggerated.)

radius in these machines. In the cyclotron, or nonrelativistic situation, the angular velocity ω is constant, and thus the orbital radius r increases directly with the velocity V, since $V=r\omega$.

In the Type I machine, however, ω decreases and V increases. This causes an even larger increase in the radius r. Thus, both the cyclotron and the F-M cyclotron require a complete, evacuated "pill-box" chamber and a solid, circular, uniform-field-producing electromagnet as shown in Fig. 2. The 184-inch machine at Berkeley for accelerating heavy particles is of this type.

In the Type II machine, the magnetic field is varied so as to keep the angular velocity ω constant. Thus, r increases directly with V. A synchrotron of this type for electrons having low injection energies would also require the above type of construction plus laminations. However, for injection energies above 2–3 Mev, the electron velocity is already very nearly equal to the speed of light and cannot change appreciably, and the orbital radius is then practically constant. This leads to one of the best schemes for accelerating electrons.

THE TYPE II-B MACHINE

This is the scheme which starts out as a betatron and then switches over to Type II operation. The particle is initially both accelerated and guided by the betatron action of the magnetic field. The switch-over is accomplished by putting an early-saturating material in the central portion of the magnet. A radiofrequency electric field is turned on when the central core saturates. At this point the betatron accelerating action ceases while the flux density at the orbit continues to increase in time. Thus, the particle's acceleration is continued by synchrotron action. Since the betatron action is capable of accelerating the electron into the extreme relativistic region, the electron velocity at switch-over is essentially that of the speed of light. Thereafter, the orbital radius does not vary appreciably from the stable betatron radius, making the "doughnut" vacuum chamber construction usable. As shown in Fig. 3, the construction is essentially that of the betatron. The writer suggests the name "synchro-betatron."

A 70-Mev electron accelerator of this type has been built successfully at the General Electric Company by Pollock and others.[11]

Electron accelerators of this type have also been built at Cornell[12] and Massachusetts Institute of Technology,[13] where energies in the 300-Mev range have been achieved.

THE SYNCHROTRON (TYPE III)

The third type is the most complicated machine of the three since it involves simultaneous variation of both the frequency of the electric field and the intensity of the magnetic field. In principle, it is essentially a Type II machine wherein the radiofrequency is increased in such a manner as to maintain a constant orbital radius while the particle's energy is increasing. This permits the use of a doughnut vacuum chamber and a hollow-centered magnet, as shown at the right in Fig. 4.

Crane[14] at the University of Michigan has proposed this scheme for a 200-Mev electron accelerator. Oliphant *et al.*[15] proposed this for the 1-Bev Birminhgam proton accelerator, and the same principle will be employed in the Brookhaven[16] "Cosmotron" and the Berkeley "Bevatron."

The operating principle of this machine differs from that of the Type II machine in the following manner. When the particle is gaining energy still traveling at nonrelativistic velocities, the magnetic field is not permitted to remain constant. Instead, it is increased in order to increase the angular velocity in such a manner that the orbital radius remains constant ($r=V/\omega$). At the same time the radiofrequency

[11] H. C. Pollock, Phys. Rev. **69**, 125 (1946).
[12] McDaniell, Dewire, Corson, and Wilson, Phys. Rev. **76**, 162 (1949).
[13] N. H. Frank, Phys. Rev. **70**, 177 (1946).
[14] H. R. Crane, Phys. Rev. **69**, 542 (1946).
[15] Oliphant, Gooden, and Hide, Proc. Phys. Soc. (London) **59**, 666 (1947).
[16] Ralph Livingston, Phys. Rev. **73**, 1258 (1948).

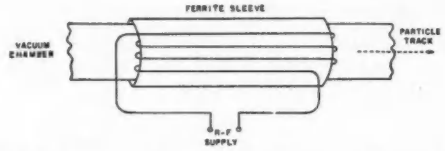

FIG. 5. Wide-band radiofrequency induction accelerator for Racetrack synchrotons. The alternating flux in the ferrite sleeve produces an alternating electric field which is coaxial with the straight portion of the particle track.

is increased in synchronism with this angular velocity so that the particle continues to gain in energy. In the extreme relativistic condition, the particle velocity does not change appreciably. In this region the radiofrequency is not varied appreciably, and the magnetic field continues to increase in the manner of the Type II machine.

The exact manner in which the radiofrequency should increase in correspondence with the variation of the magnetic field requires very critical control, according to Gooden *et al*,[17] if any appreciable yield of high energy particles is to be expected. The range of frequency variation is about 30 to 1 for the Birmingham machine.

THE RACETRACK SYNCHROTRON (TYPE IIIA)

A further extension of Type III which should be described here is the "Racetrack," a further suggestion of Crane.[18] This is the same as the Type III machine except that the magnet is broken up into two semicircles[19] separated by straight sections free of magnetic field wherein the acceleration is accomplished by the action of the radiofrequency electric field. The plan of this machine is shown in Fig. 4.

The Racetrack probably represents the ultimate in the design of particle accelerators since it is a complex composite of the cyclotron, the betatron, and the linear accelerator. The objection usually raised against linear accelerators is that they require a large number of accelerating sections in order to impart very high energies to particles. In the Racetrack, however, a few linear accelerating sections are used again and again. Also, the problems of injection and removal of the particles are simpler in the racetrack than in the circular designs.

The Brookhaven cosmotron is of the Racetrack variety.[20] The name fits this colossus with no stretch of the imagination, since it is to be a racetrack not only in shape but in size as well! The orbital diameter is of the order of 50 feet. The solution of the high vacuum problem in this machine presents a tremendous challenge.

In the Brookhaven machine the high frequency electric field is to be established not by electrostatic gaps but by electromagnetic induction.[21] A concentric alternating flux will be produced in a powdered iron pipe which encircles a straight section of the track. This is diagrammed in Fig. 5. The flux will be produced by a coil winding which can be driven by a radiofrequency signal source over a wide range of frequency (approximately 300 kc to 4.3 Mc). It is planned to make the frequency variation self-adjusting through an automatic control system and a series of particle-energy monitors. It was, originally hoped that 3-Bev protons or 6-Bev alpha-particles would be produced.

Thus far the discussion has been confined to describing the essential features and the central principles of operation of these machines in the ideal case. Next, attention will be given to those aspects of the design which affect any departures from ideal operation.

PHASE OSCILLATIONS

Phase oscillations have been studied by a great number of workers.[1,2,5,13,22] The phase of a particle is its angular position at the time when the gap voltage passes through zero from positive to negative acceleration. As mentioned in earlier paragraphs, the stable condition for a relativistic particle is near zero phase. Actually, the particle continually oscillates about the point of stable phase. These oscillations are slow with respect to the time it takes for the particle to make one revolution. If this oscillation carries the particle beyond $\pm 90°$ in phase, the particle will fall out of synchronism with the accelerating field.

[17] Gooden, Jensen, and Symonds, Proc. Phys. Soc. (London) **59**, 677 (1947).
[18] H. R. Crane, Phys. Rev. **70**, 800 (1946).
[19] Four quadrants have also been proposed. See D. M. Dennison and T. Berlin, Phys. Rev. **70**, 174 (1946).
[20] J. R. White, Phys. Rev. **75**, 1288 (1949).
[21] T. Plotkin and J. P. Blewett, Phys. Rev. **75**, 1288 (1949).
[22] D. M. Dennison and T. Berlin, Phys. Rev. **70**, 174 (1946).

The differential equation for the phase motion which results from a consideration of the restoring forces and damping forces acting on the particle appears similar to the equation of a pendulum.[1] Just as in the pendulum, the restoring force diminishes when the angle exceeds 90°. Equations of this sort led to the relations of Veksler mentioned previously which are the conditions necessary for phase stability.

Gooden, Jensen, and Symonds list no fewer than eight factors which affect these oscillations in the Type III machine.[16]

Built-in Factors

1. The restoring force constant depends on the instantaneous phase position of the particle.
2. The small betatron damping effect varies with radial position.
3. The restoring force constant depends on the energy of the particle.
4. The restoring force is affected by the radial position of the particle.

Adjustable Factors

5. Rate of rise of the radiofrequency.
6. Amplitude of the rf voltage.
7. Rate of rise of the magnetic field.
8. Shape of the accelerating electrodes.

RADIAL AND VERTICAL FOCUSING

In all forms of Type III machine, radial and vertical focusing is accomplished by shaping the pole faces of the magnet as shown in Fig. 6. According to Kerst and Serber[23] it is necessary that H vary as $1/r^n$ where $0<n<1$. The value $n=\frac{2}{3}$ has been used successfully.

This variation is necessary because the centrifugal or inertial force acting on a particle having a velocity V is mV^2/r, if the motion is circular. This force decreases as $1/r$. The centripetal force Hev will decrease less rapidly with r if H varies in the manner indicated above. Thus, a particle displaced radially from its equilibrium orbit will experience unbalanced forces which will restore it to its original orbit.

The shape of the pole faces also provides vertical focusing. Above the median plane, it can be seen from Fig. 6 that a horizontal component of H will act on the moving particle and force it back toward the median plane.

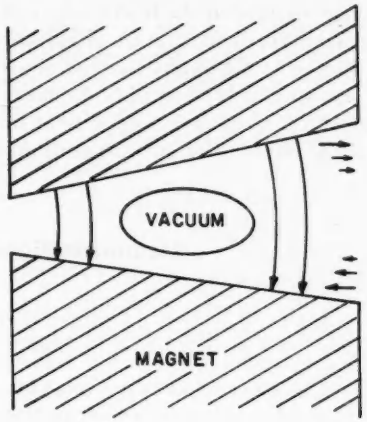

FIG. 6. Magnetic-field shaping in the betatron and the synchrotron which provides focusing in the radial and vertical directions.

Equations describing the radial and vertical motions of a particle which has been initially displaced from its equilibrium orbit have been written.[17,23] In this way, the factors affecting the oscillation of a particle in these directions have been investigated.

One factor responsible for initiating orbital oscillation is the process of injection. The particles cannot all be shot into the exact position of the stable orbit. The proper focusing forces must exist to pull the particles into a stable orbit, along with sufficient damping action to keep the orbital oscillations small so that the particles are not all lost by hitting the sides of the vacuum chamber.

The phase oscillations also contribute to radial oscillations since oscillations in phase coincide with oscillations in particle energy, which in turn cause oscillations in orbital radius.[17]

For the Racetrack having two straight sections Serber[24] has pointed out that there is a theoretical limit to the ratio of length-of-straight-section to orbital-diameter for a given shaping of the magnetic field. Dennison and Berlin[19] have pointed out additional restrictions on the shape

[23] D. W. Kerst and R. Serber, Phys. Rev. 60, 53 (1941).

[24] R. Serber, Phys. Rev. 70, 434 (1946).

of the magnetic field in the Racetrack, and they have carried Serber's idea on to an investigation of orbital stability when any number of straight sections are employed. For the aforementioned ratio equal to 0.3 and a magnetic field shaping of $n=\frac{2}{3}$ they found that an optimum stability exists when four straight sections are employed. The Racetracks at Brookhaven, Berkeley, California Institute of Technology, and the University of Michigan are all of the four-sided variety, consisting of four guiding quadrants separated by four short accelerating straight sections.

Demonstrations with Half-Wave Plates*

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University of Wisconsin, Madison, Wisconsin
(Received August 14, 1952)

Cellophane of thickness to give 180° phase difference between *O* and *E* beams may be combined with polarizing sheets to make three types of windows. A design is cut in the Cellophane and it is mounted on the polarizer with effective optic axis 45° to the plane of vibration transmitted. The three types are (a) linear polarizer with half-wave Cellophane (b) linear/circular polarizer with half-wave Cellophane mounted on the side opposite the quarter-wave plate (c) linear/circular polarizer with half-wave Cellophane mounted on the same side as the quarter-wave plate.

Two windows of type (a) give eight different patterns depending on the arrangement. One window of type (a) permits detection of small degrees of polarization. With one window of type (b) and one of type (c) there are six possible patterns two of which do not change with rotation of one window with respect to the other. These effects result from the ability of a half-wave plate to rotate the plane of vibration through any angle or reverse the sense of circularly polarized light.

THE double refraction of Cellophane can be used to provide a number of demonstrations of some properties of polarized light. Most Cellophane is biaxial[1,2] like mica[3] with optic axes as shown in Fig. 1. For parallel light along

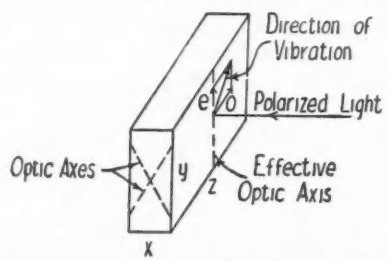


FIG. 1. Relation of real optic axes to effective optic axis for Cellophane or mica.

the normal to the surface, the incident beam, the two optic axes, and the *O* and *E* beams all lie in one principal plane. As a result, Cellophane and mica act toward parallel light travelling in the direction of the normal as though there were one effective optic axis lying in the plane of the surface. Fairly uniform Cellophane of good optical quality with thickness to give one-half wave or 180° separation between *O* and *E* beams can be found on commercial packages. There is some but not much change of phase difference with wavelength so that when visible light passes through half-wave Cellophane and the *O* and *E* beams recombine to form plane-polarized light, the light will not be completely extinguished by a polarizing sheet. There will be produced a deep purple color at the position for extinction.

With two sheets of half-wave Cellophane and two sheets of polarizing material one may produce two windows which will show eight different patterns of light and dark designs depending upon the way in which the light goes through the windows and on whether or not one window is rotated 90° with respect to the other. Various designs are cut in the two sheets of half-wave Cellophane and each sheet is mounted on a

* Reported in part to American Association of Physics Teachers, Chicago, Illinois, October, 1951.
[1] P. G. Drummond, Nature **145**, 67 (1940).
[2] R. C. Gray, Nature **145**, 266 (1940).
[3] F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill Book Company, Inc., New York, 1950), 2nd edition, p. 530.

polarizing sheet with the effective optic axis at an angle of 45° to the plane of vibration transmitted by the polarizer. The eight patterns obtained with two such windows are shown in Fig. 2.

The effect of the Cellophane in these windows is to rotate the plane of vibration of polarized light by 90°. The 90° rotation of the plane of vibration by a half-wave plate results as shown in Fig. 3, A and B. The polarized light as it enters the Cellophane is equivalent to *O* and *E* beams with phase difference zero. If the Cellophane introduces a phase difference of 180° between the *O* and *E* beams and if the amplitudes of the two beams are equal, they combine on emergence to give plane polarized light with a direction of vibration 90° from that of the polarized light which entered. With polarizer and analyzer crossed as in Fig. 3, light which passes through the Cellophane will pass through the analyzer while light which passes through a hole in the Cellophane will be absorbed. The observer sees a black design surrounded by a white background (Fig. 2h). Rotation of analyzer by 90° changes the pattern to a white design and black background or a change from a negative to a positive pattern, Fig. 2g. This is shown also in Fig. 2, e and f.

If the light goes through a second sheet of Cellophane before reaching the analyzer there is introduced a second 180° phase difference between *E* and *O* beams and if the second Cellophane is oriented to give equal intensities for the *E* and *O* beams, there results a total rotation of 180° or 0° for the plane of vibration. This produces polarized light vibrating in the direction of the light as it entered the first Cellophane sheet. If there is also a design cut in the second sheet of Cellophane and the polarizer and analyzer are crossed, the light which goes through two cut-out places where there is no Cellophane, is absorbed by the analyzer as shown in Fig. 2c. At places where the light goes through two sheets of Cellophane it is also absorbed. At places where the light goes through just one sheet, it is transmitted through the analyzer. Under these circumstances if one window is moved without rotation with respect to the other so that two white places overlap they will become black, as in Fig. 2c. If one window (Cellophane plus polarizer) is rotated 90° with respect to the other, a different pattern is obtained. If now one window is moved with respect to the other so that two dark places overlap they will become light, as in Fig. 2d.

When the two windows are so arranged that there is no Cellophane between the two polarizing sheets, the Cellophane produces no observable effect and the two windows behave like ordinary Polaroid sheets. Fig. 2a and b. The polarized light that enters the eye through the Cellophane is of course rotated 90° but the eye does not detect this. On the other side, the unpolarized light passing through Cellophane into a polarizer is not changed with respect to polarization by the Cellophane.

With the arrangement as shown in Fig. 3 and the Cellophane fastened to the polarizer, if the analyzer is rotated 90° the design changes from a positive to a negative. There is a certain analyzer angle at which the design disappears and this angle can be closely fixed. If the polarizer is rotated or if a quartz crystal or sugar solution causes a rotation of the plane of vibration, the analyzer can be turned until the design again disappears and the angle of rotation of the plane of vibration determined. The appearance is like that obtained with a double image prism when the two images are viewed through a polarizing window. For the accurate location of the plane of vibration or the accurate determination of the angle through which the plane of vibration has been rotated, photocells may be used to compare the intensities of light coming through the half-wave Cellophane with that through the part cut out of the Cellophane, as done with double image prisms.[4,5]

PARTIALLY POLARIZED LIGHT

Another use for one window made of a polarizer and half-wave Cellophane with a design cut out of the Cellophane is for the detection of partially polarized light. If partly polarized light goes through the half-wave Cellophane first and then through the Polaroid sheet, a rotation of the window causes the design to change from positive to negative. This change in design can be de-

[4] L. R. Ingersoll, Phys. Rev. 9, 259 (1917).
[5] L. R. Ingersoll, ONR. Research Reviews, June, 1952, p. 16.

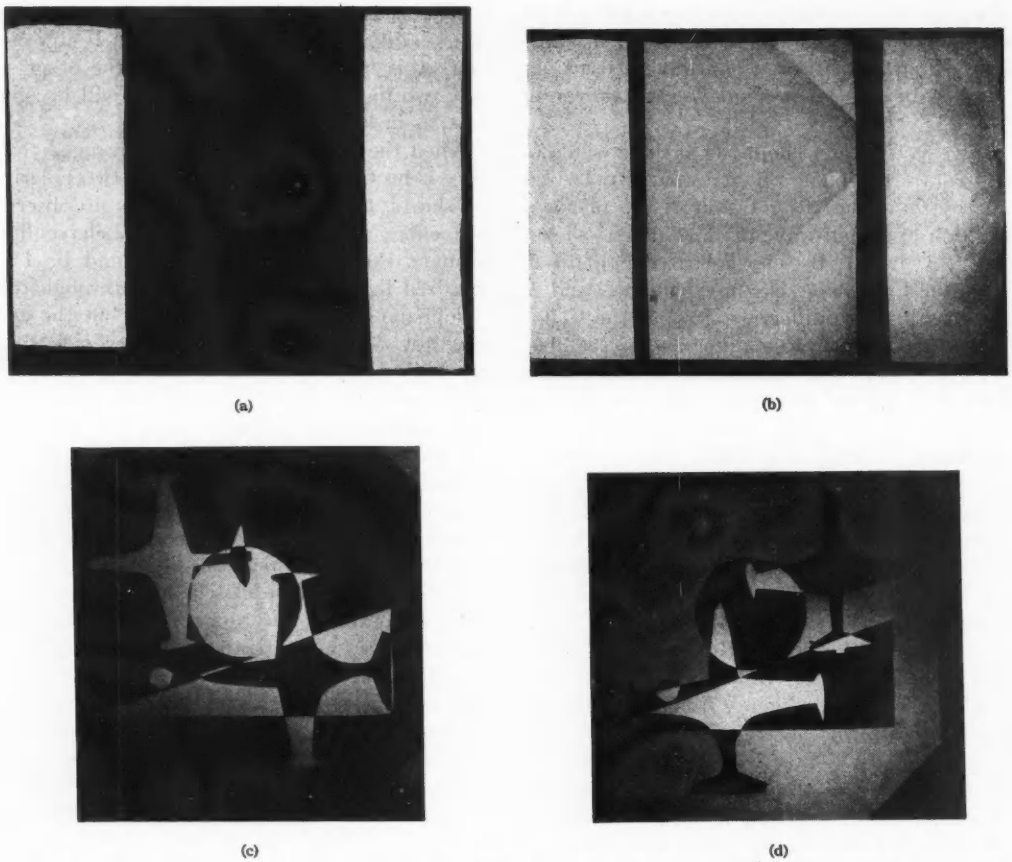


FIG. 2. Patterns obtained by arrangements of two windows each made of a polarizing sheet and sheet of half-wave Cellophane. (a) (b) No Cellophane between polarizers. (a) Polarizers perpendicular. (b) Polarizers parallel. (c) (d) Two

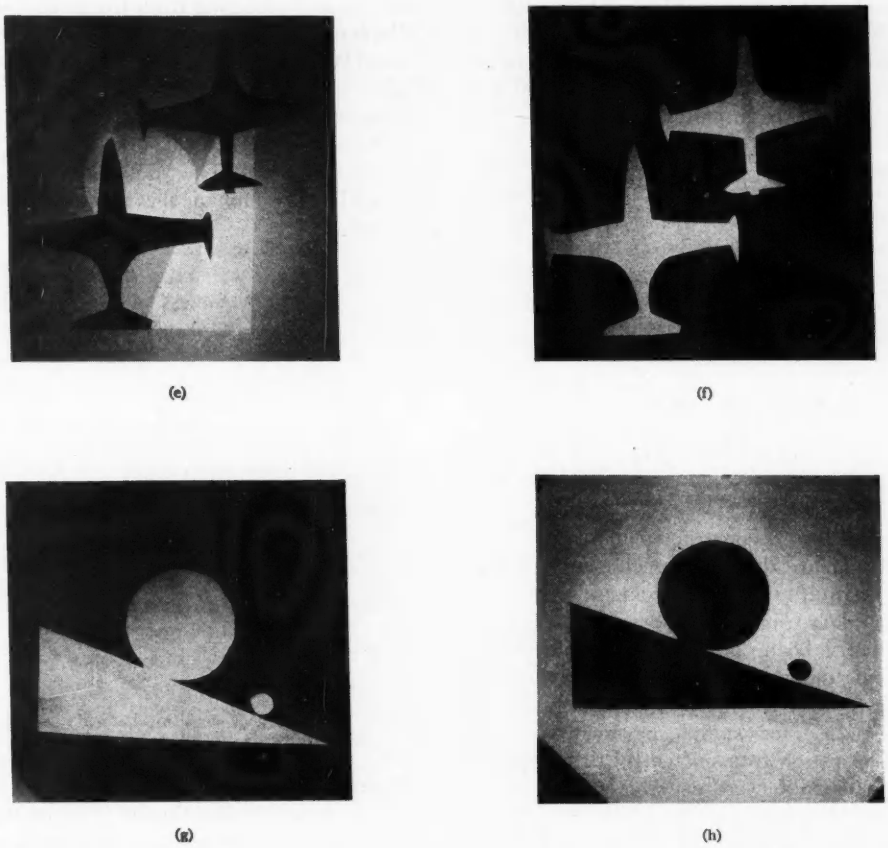
(*Caption continued on p. 173*)

tected for a small degree of polarization. A window made with a sheet of half-wave Cellophane dividing the field of view into two parts was first described by Pfund,[6] as a plate for detecting small degrees of polarization.

The property of rotation of the plane of vibration through any angle by a half-wave plate can be demonstrated by placing a sheet of half-wave Cellophane between two crossed polarizers as in Fig. 3. If the effective optic axis of the Cellophane is 45° from the plane of vibration transmitted by the polarizer, the light through the Cellophane will not pass through the analyzer. If now the Cellophane is rotated through a small angle, say 10° clockwise without disturbing either polarizer or analyzer some light will pass through the analyzer. When the analyzer is rotated through an angle of 20° clockwise the light is again extinguished. If the Cellophane is rotated through any angle θ, the plane of vibration of the polarized light is rotated through an angle 2θ. The way in which the rotation is accomplished is illustrated in Fig. 4.

By the use of two half-wave plates, a mechanical rotation through angle θ can produce a rotation of the plane of vibration through angle 3θ as shown in Fig. 5. The polarizer and half-wave plate (2) are fastened so that they rotate together through the same angle. Half-wave plate 1 is fixed in position. If the polarizer and half-wave plate 2 are rotated say 30° clockwise

[6] H. Pfund, J. Opt. Soc. Am. 26, 453 (1936).



(e) (f)

(g) (h)

Cellophane sheets between polarizers. (c) Polarizers perpendicular. (d) Polarizers parallel. (e) (f) (g) (h) One Cellophane sheet between polarizers. (e) (h) Polarizers perpendicular. (g) (f) Polarizers parallel.

the light leaving half-wave plate 1 will have been rotated 60° counterclockwise from the effective optic axis of half-wave plate 2. The light leaving plate 2 will have a plane of vibration 60° clockwise from the effective optic axis of plate 2 and will be a horizontal vibration. Thus a 30° clockwise rotation of the polarizer and half-wave plate 2 produces a 90° clockwise rotation for the emerging polarized light. This procedure could be extended with any number of half-wave plates arranged so that alternate plates rotated with the polarizer and the others remained fixed. For an even number of half-wave plates the angular multiplication factor is $+(n+1)$, where n is the number of plates. For an odd number, the multiplication factor is $-n$.

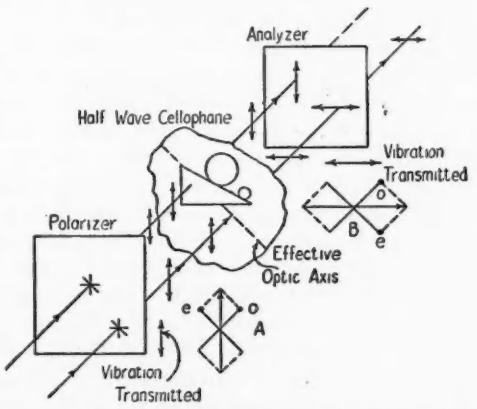


FIG. 3. Polarized light has its plane of vibration rotated 90° by a properly oriented half-wave sheet.

The number of half-wave plates which could be used with visible light would be limited by the dispersion of the Cellophane. With monochromatic light the number of plates would be limited only by the intensity.

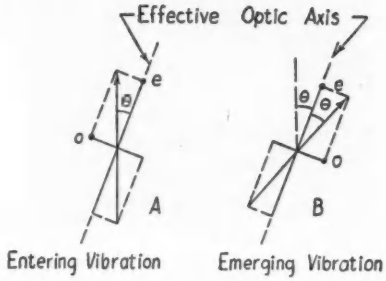


FIG. 4. Action of half-wave plate in rotating plane of vibration of polarized light through any angle.

HALF-WAVE PLATES

It is possible to construct a half-wave plate out of two identical sheets of Cellophane neither of which is quite the right thickness to give exactly 180° phase difference between ordinary and extraordinary beams. This is accomplished by first placing one sheet of Cellophane behind the polarizer so that its effective optic axis makes an angle of 22½°, for example, clockwise from the plane of vibration transmitted by the polarizer. If the Cellophane were exactly one-half wave thickness the light emerging from the Cellophane would be plane polarized light with vibrations at an angle 45° clockwise from the plane of vibration of the polarized light that entered the Cellophane. If, however, the thickness is such that it is not exactly a half-wave plate but is, say, a thickness to give 190° phase difference, there results elliptically polarized light with the major axis at angle 45° clockwise from the plane of vibration of the polarized light that entered the Cellophane. Now the second sheet of Cellophane is placed behind the first with its effective optic axis making an angle of 45° counterclockwise from the optic axis of the first sheet of Cellophane or angle 67½° counterclockwise from the major axis of the elliptically polarized light or 22½° counterclockwise from the direction of vibration of the original polarized light. The excess phase difference introduced by the first Cellophane sheet is counteracted by the second to give a 180° phase difference between the *O* and *E* beams emerging from the second sheet of Cellophane. There results plane polarized light rotated 90° from the direction of vibration of the original polarized light. A photograph of such a combination half-wave plate is shown in Fig. 6. The lower part is the place of overlap of the two Cellophane sheets. The spots are holes; some through both Cellophane sheets and some through only one.

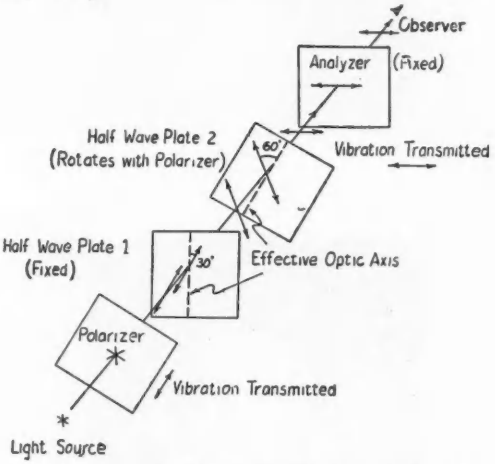


FIG. 5. Conversion of small mechanical rotation into large rotation of plane of vibration.

The property of half-wave Cellophane to rotate the plane of vibration by 90° may be of value in glasses for viewing stereoscopic motion pictures. It was pointed out by Mr. Richard T. Kriebel of the Polaroid Corporation in conversation with the writer that a half-wave plate could be used to give the members of the audience the option of seeing the motion pictures either flat or in three dimensions. All that is necessary is to place a half-wave sheet of Cellophane in front of one of the windows of the Polaroid viewing glasses so that the effective optic axis is at 45° from the direction of vibration transmitted by the polarizer. If this half-wave Cellophane is on the side toward the eyes, it will produce no effect and the observer will see three-dimensional pictures. On the other hand if the Cellophane is between the glasses and the screen the Cellophane will rotate the plane of vibration 90° for one eye. Then both eyes will see the same picture, and the picture will be seen as a flat two-dimensional picture.

Another way to accomplish the same option is to make the viewing spectacles with one of the polarizing viewing sheets loose so that it can be lifted out, rotated 90° and inserted back in place. After such rotation, motion pictures viewed through the glasses will appear flat.

LINEAR/CIRCULAR POLARIZERS

The 90° rotation produced by a half-wave plate may also be produced by two quarter-wave plates or by one quarter-wave plate through which light passes twice. The linear/circular polarizers manufactured by the Polaroid Corporation[7] consist of one polarizing sheet cemented to a quarter-wave plate with effective optic axis 45° from the plane of vibration transmitted by the polarizer. Light passing first through the polarizer emerges circularly polarized while light passing first through the quarter-wave plate emerges plane polarized. Light reflected from a mirror after passing through a linear/circular polarizer window will not return through the window if the quarter-wave plate faces the mirror. Two passages through the quarter-wave plate make the plate serve as a half-wave plate and, since the orientation is that to give a 90° rotation of the plane of vibration, the returning polarized light does not go through the Polaroid sheet. When the window is turned over so that the quarter-wave plate is away from the mirror, the quarter-wave plate produces no effect and the reflected light is transmitted.

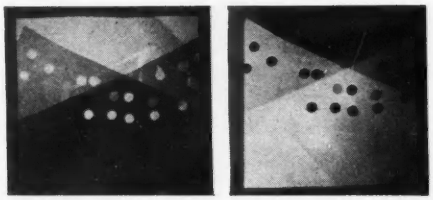


FIG. 6. Production of half-wave plate by overlap of two Cellophane sheets.

Another way to describe the effect of the mirror and the linear/circular polarizer is to recognize that the reflection from the mirror changes the circularly polarized light from say clockwise to counterclockwise as viewed along the direction of travel of the light. If a quarter-wave plate changes vertical vibrations into clockwise circularly polarized light, it changes counterclockwise polarized light into plane polarized light with vibrations horizontal. Thus the returning light does not pass through the polarizer.

Some additional unusual effects are obtained with windows made with linear/circular polarizers and half-wave Cellophane sheets. One window is made with the Cellophane containing a cut-out design, fastened to the linear/circular polarizer on the side opposite the quarter-wave plate (Fig. 7A) and another window made with the half-wave Cellophane containing another cut-out design fastened to the same side of the linear circular polarizer as the quarter-wave plate (Fig. 7B). With these two windows there are obtained six different patterns of designs as shown in Fig. 8, depending on the orientations of the windows with respect to each other. In two arrangements, Figs. 8c and 8d, one window can be rotated with respect to the other and there results no change in the pattern of light.

The action of each of these windows is shown in Figs. 7A and 7B. If light goes through window B, passing first through the half-wave and quarter-wave plates, window B acts like an ordinary sheet of polarizing material. If light goes through window B, passing first through the polarizer it will be plane polarized as it enters the quarter-wave plate and circularly polarized (for example counterclockwise) as it enters the half-wave plate. The half-wave plate converts the circularly polarized light from counterclock-

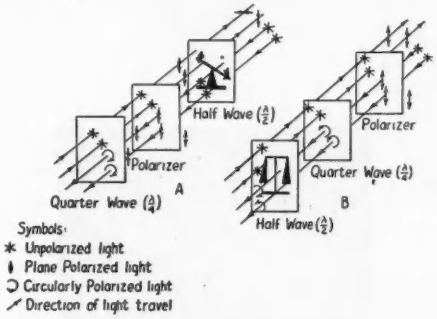


FIG. 7. Action of windows made of linear/circular polarizers and half-wave sheets.

[7] Polaroid Reporter, No. 1, 1951.

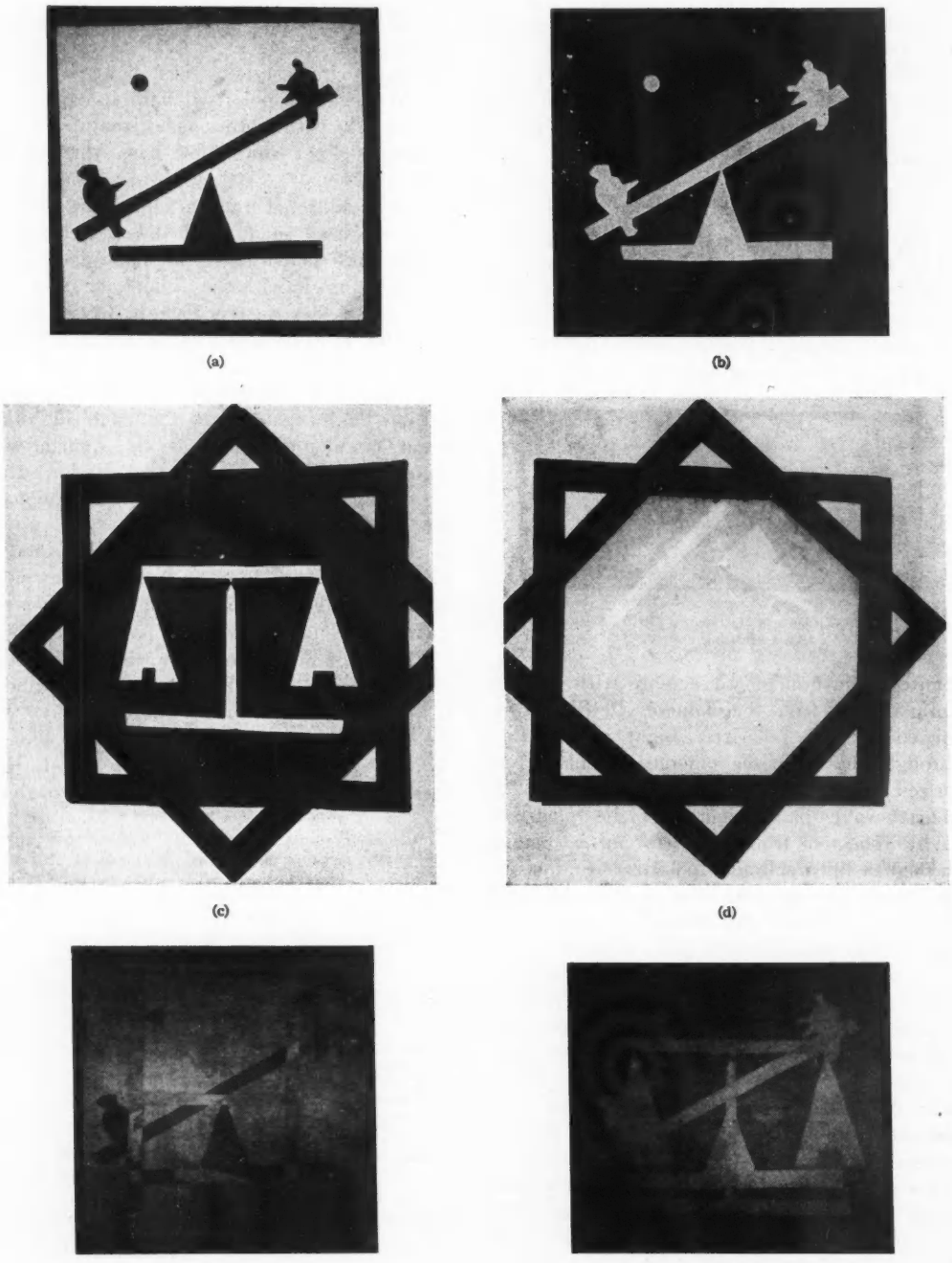


FIG. 8. Patterns obtained with various arrangements of two windows each made of linear/circular polarizer and half-wave Cellophane. (a) (b) One half-wave sheet between polarizers. (a) Polarizers perpendicular. (b) Polarizers parallel. (c) One half-wave and two quarter-wave sheets between polarizers. (d) One quarter-wave sheet between polarizers. (e) (f) Two half-wave and one quarter-wave sheets between polarizers. (e) Polarizers perpendicular. (f) Polarizers parallel.

wise to clockwise. Thus light that goes through the background in the half-wave plate will be clockwise while that which goes through the cut-out design will be counterclockwise circularly polarized.

For window A, if unpolarized light goes first through the quarter-wave plate it emerges from the polarizer as plane-polarized light. Then that part which goes through the background half-wave plate is rotated 90° from that part that goes through the cut-out design. If for window A the light goes first through the half-wave plate, the half-wave plate produces no effect and the light emerges from the polarizer as plane polarized light. This polarized light passes through the quarter-wave plate to become circularly polarized say clockwise over the entire surface of the window.

One of the patterns (Fig. 8c) which does not change with rotation of one window with respect to the other was obtained with the arrangement shown in Fig. 9 with the light going as shown. If window B is turned over the pattern of Fig. 8d is obtained. The patterns of Figs. 8a and b are obtained by interchanging windows A and B from the case shown in Fig. 9.

To obtain the patterns of Figs. 8e and 8f window A is turned over and window B left as shown in Fig. 9. This puts two half-wave plates and one quarter-wave plate between polarizing sheets. Window B produces circularly polarized beams some of which have their sense of rotation reversed by the half-wave plate on window A. The result is still circularly polarized light entering the polarizer of window A. If there were no dispersion, that is if the windows were quarter- or half-wave plates for all wavelengths, there would be no change in a pattern of uniform brightness as one window was rotated. Because of dispersion, however, there results colored designs and backgrounds which change with rotation of one window.

If with the arrangement of Fig. 9 the half-wave plate of window B is not fastened to the linear/circular polarizer, it is observed that rotation of the half-wave plate alone leaving everything else fixed produces no change in the pattern. Any rotation (half-wave plate, linear/circular polarizer, or window A) gives a rotation

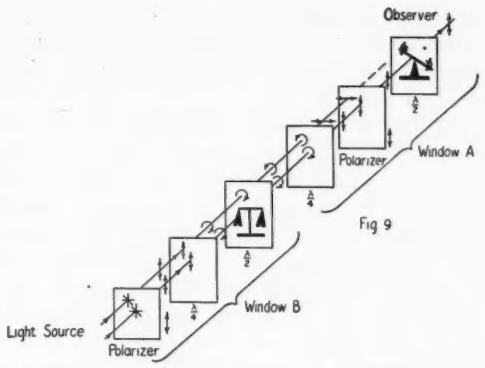


FIG. 9. Arrangement of windows to obtain Fig. 8c.

with respect to circularly polarized light and produces no change in pattern.

INTERFERENCE EFFECTS

The arrangement of a doubly refracting sample between two linear/circular polarizers with quarter-wave plates facing the sample is one shown to be useful in studying stress patterns by M. Hetenyi.[8] The circularly polarized light which goes through the sample is resolved into *O* and *E* beams of equal amplitude regardless of the direction of the optic axis of the sample. A phase difference between the *O* and *E* beams which depends on the thickness and stress in the sample is introduced. These *O* and *E* beams then pass through the second quarter-wave plate and polarizer to emerge as two equal amplitude beams of plane polarized light vibrating in the same direction. These produce an interference which depends only on the phase difference introduced by the sample. The interference pattern obtained is independent of the orientation of the optic axes in the sample, and does not change with rotation of either linear/circular polarizer.

If the sample is placed between ordinary Polaroid sheets there is observed a pattern of interference, due to phase differences, crossed by dark lines which change position in the sample as the sample is rotated. These dark lines denoted as isoclinics in photoelastic analyses and isogyres in crystal optics occur at places where

[8] Address to American Association of Physics Teachers, Chicago, Illinois, October 26, 1951, and Photo Technique (November, 1940); J. Appl. Phys. **10**, 295 (1939).

the optic axis of the sample is either parallel or perpendicular to the direction of vibration of the plane polarized light passing through the sample. At these places the polarization of the light is not affected by the sample regardless of the stress or thickness. The light through these places is absorbed by the analyzer and the observer sees dark lines over the surface of the sample. When the sample is rotated these dark lines may rotate with an angular velocity different from that of the sample. All of the isoclinics or isogyres disappear when linear/circular polarizers are used as described above.

If with any of the arrangements described above, any of the components (half-wave plate, combination window, or two combination windows together) are tipped, i.e., rotated about an axis in the plane of the surface, there generally results a change in the color of the light through the Cellophane. The change observed depends on the direction of the axis of rotation with respect to the direction of the effective optic axes. Suppose two Polaroid half-wave Cellophane windows are arranged to give a pattern like that in Figs. 2f or 2g, with dark purple background. If one window is tipped about the effective optic axis as an axis of rotation the background becomes blue-green. If the window is tipped about an axis perpendicular to the effective optic axis the background becomes brown. With windows arranged to give Fig. 8e, the pattern may be changed from a positive to a negative by tipping.

Tipping about the effective optic axis as an axis of rotation increases the optical path for both *O* and *E* beams and increases the phase difference between them. If, before tipping, the phase difference was 180° for green light it was less than 180° for red and more than 180° for blue. Green light then had its plane of vibration rotated 90° and was absorbed in the analyzer but some red and blue got through to give the dark purple color observed. After tipping, because of the increase in phase difference, red light has a phase difference 180°, is rotated 90°, and absorbed in the analyzer. Some blue and green then get through the analyzer to give the blue-green color observed.

Tipping about an axis of rotation perpendicular to the effective optic axis causes light to pass through the half-wave Cellophane more nearly in the direction of one of the real optic axes (Fig. 1), and the phase difference between *O* and *E* beams is reduced. When the phase difference for blue light is reduced to 180°, the blue light is rotated 90° and absorbed in the analyzer. Some red and yellow get through to give the brown color observed.

The index of refraction is not the same for all Cellophanes but has an average value $n_e = 1.535$ and $n_o = 1.521$ with the angle between the real optic axes and the normal to the surface 46°.[9] This angle exceeds the critical angle for Cellophane in air. It is thus not possible to tip Cellophane in air enough for light to pass through along a real optic axis.

If mica is placed between crossed polarizers and tipped, the interference pattern disappears at one angle of tip and then re-appears at larger angles. For mica for which n_o is 1.59 and the angle between the real optic axes and the normal to the surface is 21°, light will pass through along an optic axis for an angle of incidence of 35°. At this angle of incidence the interference pattern disappears. Tipping mica about an axis of rotation parallel to the effective optic axis gives an interference pattern for all angles of incidence from 0 to 90°.

HALF-WAVE CELLOPHANE WINDOWS

In constructing a combination half-wave Cellophane-polarizing sheet-window, Pfund[6] used linseed oil surrounding the Cellophane to reduce the visibility of the edges. The use of linseed oil, still leaves the outline of the design cut in the Cellophane quite visible. The use of tung oil, however, completely eliminates the visibility of the edge of the Cellophane. Tung oil has an index of refraction 1.515–1.520 which is sufficiently close to the average 1.528 of Cellophane to give extinction. The use of tung oil, on both sides of the Cellophane and polarizing sheet and on the inside surfaces of the cover glasses has the additional effect of eliminating reflections from inside surfaces. The windows should not be bound with masking tape since some adhesive diffuses into the tung oil and destroys the polarization. Ordinary Scotch tape is satisfactory.

A permanent seal and invisible design may be made with Canada balsam (refractive index

[9] D. G. Drummond, Nature **145**, 67 (1940).

1.530) as done by Wright[10] in his experiments using mica sheets. Dried Canada balsam, dissolved in benzene forms a viscous solution which adheres to glass and in which Cellophane is invisible. The window is allowed to stand several days and is then baked in an oven for about 15 minutes at a temperature about 150°F with a small weight on the window. When the window cools it should be found free of bubbles.

A permanent seal with invisible design may also be made with polyvinol acetate.

A small sheet of half-wave Cellophane mounted in Canada balsam between glass plates could serve as a beam splitter or replace a Wollaston prism in some of the optical instruments described by R. Clark Jones and C. D. West[11] and W. R. Brode.[12] A Cellophane beam splitter would give two beams with no divergence between them.

Many of the very interesting demonstrations with polarized light described by Wright[13] can now be more conveniently performed by the use of Cellophane sheets instead of the mica or selenite (transparent gypsum) used by Wright.

Kodapak sheets giving small retardations have been used by Blodgett[14] to produce a stepgauge for the determination of film thicknesses by comparing the reflection interference colors from films with the transmission colors of the stepgauge.

In conclusion it has been shown that half-wave sheets of ordinary Cellophane can be used with linear and linear/circular polarizers to make windows that show patterns of light and dark which depend on the arrangement of the windows. Half-wave plates may also be used to detect small amounts of partial polarization, fix the angle of vibration closely, or cause a large rotation of the plane of vibration by a small mechanical rotation. The effects result from the ability of half-wave plates to rotate the plane of vibration of polarized light through any angle or to reverse the sense of rotation of circularly polarized light.

The writer wishes to thank Mr. Richard Kreibel and Dr. C. D. West for a number of helpful suggestions and references. This work was supported in part by a grant from the Wisconsin Alumni Research Foundation.

[10] Lewis Wright, *Light, a Course of Experimental Optics* (Macmillan and Company, Ltd., London, 1882), pp. 258–352.

[11] R. Clark Jones and C. D. West, J. Opt. Soc. Am. **41**, 983 (1951).

[12] Wallace R. Brode, J. Opt. Soc. Am. **41**, 992 (1951).

[13] See reference 10, p. 264.

[14] Katharine Blodgett, Science **115**, 515 (May 9, 1952).

Colloquium of College Physicists

The Colloquium of College Physicists at the University of Iowa will be held June 17–20, 1953. A special feature of the Colloquium will be a series of four lectures by Dr. E. U. Condon, President of the American Association for the Advancement of Science, on the general subject, "The Physics of the Glassy State." The annual exhibit of new devices, both experimental and nonexperimental, and also of recent publications will be held. Prizes will be offered in connection with the exhibits.

Sustained Foucault Pendulums

R. Stuart Mackay
University of California, Berkeley 4, California
(Received August 6, 1952)

The time-delay or phase-shift effects crucial to the operation of ordinary doorbells and buzzers can be employed to sustain a Foucault (or other) pendulum in a very simple manner, not involving any electrical contacts. Some new electrical effects as well as the pendulum itself are discussed.

THERE are no forces to rotate the plane of oscillation of a simple pendulum and thus its plane of oscillation will remain fixed even though its support be rotated. The plane of a pendulum on the surface of the earth thus appears to rotate with respect to its surroundings as the earth rotates (rotates the surroundings). It is fairly obvious that, at the poles, the earth will rotate under the pendulum once in 24 hours; at the equator the plane of oscillation will remain fixed; and at intermediate latitudes the process will take times intermediate between 24 hours and an infinite period.

This device not only has historical significance as a direct proof that the earth rotates, but it also makes an excellent demonstration that can be left on display to arouse interest. The only problem is to maintain motion so that the effect can be observed. The first consideration of this problem was given by Foucault himself.[1] It is clear that any drive mechanism must have radial symmetry so that the swing will be equally easy in all directions. There follows the description of a very simple device to accomplish this, as well as a brief discussion of some interesting related schemes.

If one energizes a coil with ac and sets an iron pendulum bob to swinging toward and away from it, changes in current will be observed due to changes in the inductance as the iron moves toward and away from the coil. The current, and attractive force appropriate to that current, will not drop instantly as the bob approaches and will not build up instantly as the iron recedes. (The above variations are, of course, superimposed upon the normal steady increase in force on approaching a magnet.) Thus the iron will "see" a stronger attractive field as it approaches the magnet than it will at the same position in space while leaving. If the circuit looks essentially inductive (has sufficient Q) this effect will maintain the pendulum in oscillation. The effect can be made stronger by placing a condenser in series with the coil to resonate it approximately at the line frequency. (With the bob removed the circuit should be slightly inductive.) Thus, by working on the side of a resonance curve one gets larger changes in current for a given change in inductance. This particular effect has been known for some time and has recently received notice twice.[2,3]

It seems appropriate to digress momentarily at this point and mention that ordinary doorbells and buzzers, whose action is almost always misrepresented, could not work if their magnetic fields were not somewhere a double-valued function of armature position. Because of springy contacts and a finite time constant L/R, the armature is somewhere accelerated more strongly while moving toward the magnet than it is decelerated while moving away. A buzzer with sufficiently rigid contacts and fast response time could receive no net energy to maintain itself, though its action could be restored by placing a diode backward across the coil; this being equivalent to having a massive springy contact that does not "break" at the same place it "makes." Thus the fundamental action of the pendulum and that of a buzzer are identical in almost all respects.

It is interesting to consider the possibility of this same piece of apparatus being suitable for

[1] *Recueil des Travaux Scientifiques de Léon Foucault* (Publié par Madame Veuve Foucault sa mère, Paris, 1878), pp. 389, 572.

[2] P. F. Bartunek, Am. J. Phys. **19**, 57 (1951).

[3] H. P. Knauss and P. R. Zilsel, Am. J. Phys. **19**, 318 (1951).

maintaining a nonmagnetic bob (e.g., aluminum or better, copper) in motion. In this case the bob would be repelled (because of induced currents) but, by acting as a shorted turn (transformer action), it would cause an increase in current on approaching. Because of this, and taking into account time lags that are augmented by the shielding of the center of the bob by its outside, the bob would feel a greater repulsive force on leaving than on approaching and this net "kick" in the direction of movement could supply net energy to make up for losses and maintain motion. The forces with nonmagnetic bobs are much smaller, and there is greater magnetic damping, and so it is quite difficult to set up such a demonstration. However, a partial check on this effect can easily be obtained by enhancing it through resonance. For the shorted turns effectively comprising a conducting bob, a coil is substituted. With the help of an external

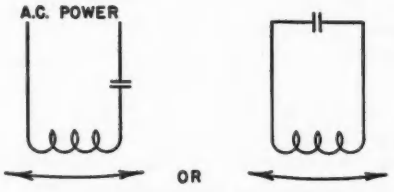


FIG. 1. If the right-hand circuit is tuned to appear inductive (similar to the case of a conducting bob), either coil will maintain the other in motion. If the pendulums are of different lengths, so they do not for long get in step and lose their driving forces, it is interesting to leave both free to swing. In radially symmetrical form this system can be used as a perpetual Foucault pendulum.

condenser the current therein is made larger. The coil might be expected to swing as a pendulum if the circuit is tuned to look inductive, i.e., like a coil. Performance of the experiment (Fig. 1) proved that as long as the right-hand circuit is inductive (C too large for resonance), the force between coils is repulsive and sustained oscillations result. Otherwise an attractive force is produced that will not sustain motion (though, because of changes in Q, in all four cases the current in the energized coil drops somewhat on approach while that in the other coil rises).

It might be pointed out that the above results make clear the observation that if the two coils are placed in series directly across the power source, sustained oscillations result. Because of the familiar rule for the total inductance of two adjacent coils, this is true no matter in which sense the two coils are connected, i.e., whether the force is repulsive and the current increases on approach, or both the opposite. As expected,

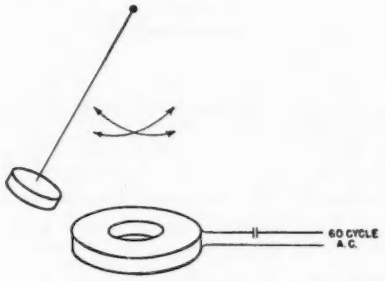


FIG. 2. A coil maintaining an iron bob swinging. The bob can swing equally easily in all directions and thus the arrangement has the symmetry necessary for a Foucault pendulum.

if the circuit includes a condenser, it must be large enough to make the net result inductive if motion is to be maintained in either case.

To maintain any pendulum, one can pull sideways on it at the end of its swing as above, or one can pull downward on it near the bottom of its path. This latter method yields the symmetry necessary to maintain a pendulum that is to swing in all directions. The arrangement is shown in Fig. 2. The pendulum feels a stronger downward and forward force as it approaches the coil than it does on leaving, and the component of this force along the path maintains motion. The response time of such a circuit is essentially the familiar Q/π cycles, and we can tell something of desirable geometry by noting that this must be comparable with the time of approach of the bob. The fact is that such an arrangement is not very critical, and hence is rather easy to make work.

The coil first tried by the author (because it happened to be readily available) was wound with number 16 wire and was about eight inches in diameter, two inches thick, and had a two-inch hole. It was found to resonate at 60 cycles with 25 microfarads and, with the application of 10 volts, drove a three-inch iron bob on the first try. Considerably more power was delivered when the magnetic field was altered by laying the coil on a six-inch circle of $\frac{1}{16}$-inch sheet iron.

A larger bob, because of the larger changes in current produced, is more strongly driven without the presence of any fixed pieces of iron. If for some reason one wishes to enhance the effect further, one can insert into the coil a cylindrical

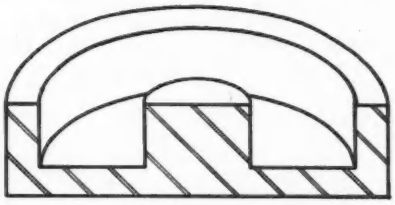


FIG. 3. Section of a core that can be inserted in the coil of Fig. 2 if it is desired to enhance the effect.

powdered-iron core such as is shown in section in Fig. 3.

This drive mechanism is interesting and will pique observers' curiosity, but it can be placed beneath the floor if desired. Sudden changes in the velocity of the bob at the bottom of the path can be obviated by using a heavier bob having more inertia. These changes actually are small because a pendulum loses only a small fraction of its total energy in one cycle (it has a high mechanical Q) and so the necessary magnetic forces, even taking into account nonvarying and useless components of the field, can be made much smaller than the gravitational ones. The lower the fields, the smaller the percentage change in downward force at the bottom of the swing, and the more nearly is the result simple harmonic motion.

In this form of the pendulum we see another interesting way of considering the case of a nonferrous bob. The armature of a single-phase induction motor is driven in whichever direction it is originally turning. Similarly a conducting bob should feel small sustaining forces whenever it passes over an energized magnet.

The comparison between a magnetic and a conducting bob might be carried a bit further. The action in the two cases is obviously the same in an ordinary pendulum where the bob is constrained to move in a plane, but now one also has the question of transverse (to the plane of motion) stability during each passage over the magnet. Over a number of cycles any small perturbing force can produce a marked effect on the swing, usually resulting in a slightly elliptical orbit. The "plane of oscillation" of a conical pendulum swinging in an elliptical path will precess in the direction of tracing the ellipse, at a rate roughly proportional to its area. Thus, if care is not taken, a Foucault pendulum can appear to turn at the wrong rate, or even indicate that the earth is turning backward. One might then inquire about the effect of passing through center a little bit to one side of the plane of the major motion. It is tempting to say that the aluminum bob will attempt to go around the "magnetic potential hill" that it "sees"; i.e., it will be deflected to the side if slightly off center, while an iron bob will tend to run through the center of the (varying-height) potential valley that it sees. Thus one might expect that if an aluminum bob is, for any reason, a little off center, on any one trip it will receive a sustained sideward push in the direction of its error, thus increasing it. Actually the exact analysis of the motion in the two cases is an interesting problem in intermediate mechanics. If a fairly heavy bob is used the above has little effect on the crosswise amplitude, and when resonances between the two directions are considered (the period being the same in all), one method seems about as desirable as the other. Interestingly enough, since the motions in the two directions are 90 degrees out of phase and the field changes are controlled by the major one, there is a slight stabilizing tendency to damp out the minor one.

Several of the planetarium museums in this country feature a display of a Foucault pendulum. They are always avidly observed by laymen and scientists alike, and hence are kept in perpetual motion by a relatively involved electronic arrangement that is based on the iron-sheathed ring-magnet shown in Fig. 4. The moving core is attached near the top of the supporting wire and as it swings through the center of its path a phototube or magnetic switch activates the magnet for a short preset interval, thus pulling the core forward in the direction it was going. Besides being a relatively complicated system, it would be subject to criticism on the previous basis except for the unusual radial dependence of its law of force. It can be considered as having a central repulsive force, most of it being exerted

near the fixed iron pole piece at the end of the throw where the ellipse and straight line orbits differ little; there being no force at the center.

The matter of perturbing effects warrants further discussion since a small one can cause difficulty when its effect is added cycle after cycle. A comparison of the two methods of magnetic drive might seem to indicate that driving a pendulum at its top would result in minimum sensitivity to asymmetries, but this is not strictly true. A pendulum is less sensitive to asymmetries at its top but similarly it requires a greater driving force there; so it is the percentage asymmetry that is of interest. An air-core coil is especially easy to construct, align, and if necessary, trim up with small metal tabs. It must be said that this simplified drive method does suffer somewhat by comparison in one respect, in that the whole field is not turned on and off to provide useful drive. Specifically, the steady useless component steadily adds in its asymmetry. To this extent the magnet must be made more perfect than in a system in which the field changes the required amount by going fully off. Of course, if one is willing to use a switching arrangement, one can devise systems that use no external field, e.g., magnetically raising and lowering a small mass inside the bob, or periodically raising and lowering the support a short distance, in synchronism with the swing.

Finally it must be stated that the task of setting up a Foucault pendulum is a tedious one. One must achieve absolute symmetry if it is to turn at the correct rate rather than seeking out a preferred direction. Besides the drive mechanism, special care must also be given the clamping of the wire whose bending constitutes the pivot. Absolute perfection can be marred even by the crystal structure of the supporting steel wire. For a demonstration such precision is unnecessary, though certain plastics might provide a simple isotropic support. With reasonable care in alignment the angular velocity should fall within 15 percent of the proper value.

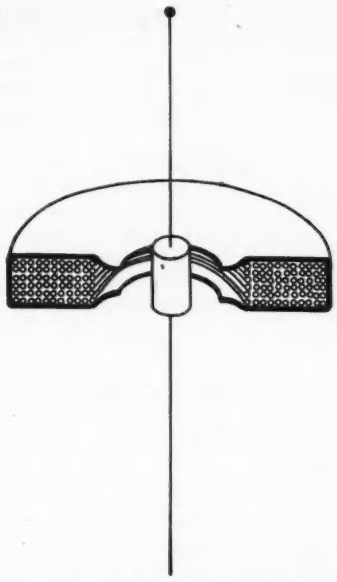


FIG. 4. Iron clad ring-magnet used to maintain a Foucault pendulum. It is electronically energized for a brief (fixed) interval each time the core passes the center.

It is felt that the above will serve as a very simple, uncomplicated and hence reliable, demonstration where achieving the precise rate of turn is quite unnecessary. It is simple enough so that it pays to have an extra smaller pendulum, set up with a rotatable support, to show the principle.

Thanks are to be expressed to Mr. Bart Parker for his great help in setting up the displays.

In one sense, perhaps the technical sense, the life of an adventurer is the practice of the art of the impossible, reserving the word hero . . . to his rationalized, moralized myth. By simple conclusion, then, heroics is in the last resort the practice of the impossible. The capture of the impregnable, the ascent of the unscalable, the logic of the illogical; wild sounding phrases, exactly, for that is just what adventure is.—WILLIAM BOLITHO.

Advances in Knowledge of Nuclear Forces

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The paper is primarily a review article covering work within the last five years preceding the summer of 1952. The principal features of experimental results on nucleon-nucleon scattering are summarized. The main conclusions of the theory of nuclear forces are reviewed with an attempt at a distinction between knowledge and speculation. The photoelectric effect of the deuteron, evidence regarding isobaric states from meson physics, the proof of the pseudoscalar character of the π-meson are briefly discussed as part of the main problem, and a partial account is given of relevant progress in the fields of binding energies of light nuclei and shell theory.

INTRODUCTION

ALTHOUGH the knowledge of the true nature of nuclear forces is still in a very rudimentary stage, such marked progress has been made in the field within the last few years that considerable clarification of the problem may be expected before long. Until recently the only adequate sources of information have been a few types of experiments on the scattering of nucleons by each other and the general features of binding energies—especially those of the lighter nuclei. The experimental material has now been extended to new ranges of energies for the nucleon-nucleon scattering, and furthermore additional evidence has come into existence in such fields as the production of mesons in nucleon-nucleon collisions under controlled conditions, advances in the knowledge of types of mesons which can be surmised to have something to do with the original Yukawa idea, the proof that the π^0 meson is pseudoscalar, evidence regarding equality of $n-p$ and $n-n$ forces from meson experiments, the strong likelihood that the interaction Hamiltonian representing the production and absorption of mesons is symmetric in isotopic spin space, etc. In addition the large progress in the classification of nuclear levels according to shell structure ideas has shed new light on the nature of the spin dependence of the interaction and has indicated the presence of $j-j$ coupling on the heavier nuclei. It should also be mentioned here that judicious handling of the dangerous inconsistencies caused by divergence troubles in meson theory by Lévy and independently by Lepore is giving hope of a promising link between the field theoretic and the purely phenomenologic approaches to the subject. Evidence for the existence of exchange moments which has been pointed out and studied by Sachs and collaborators is contributing to the hope of obtaining a still different view of the nucleon-nucleon interaction based more on their effects on each other than on the internuclear force. These exchange phenomena are related to the question of how one should interpret the existence of a quadrupole moment in the deuteron. The work of Villars has been especially illuminating in this connection because it indicates that the older phenomenological interpretation of the quadrupole moment, as a direct effect of the geometry of the neutron-proton system, is not the only possibility coming under consideration. Wigner (unpublished) has independently been pointing out that since mesons have spin 1 there is a possibility that not all of the quadrupole moment is caused by the orbital motion of nucleons. Nuclear force parameters derived from standard data on scattering combined with the quadrupole moment information should be taken with a grain of salt.

It has been realized for many years that it may not be justifiable to use a two-body type of interaction in nuclei containing more than two particles. For it is well known that, after elimination of meson field variables, the meson theories of nuclear forces give terms in the energy which depend on the arrangements of three or four particles with respect to each other and which cannot always be resolved into sums of interactions between pairs of particles. While this

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approach has not led to quantitative conclusions it has contributed to the general progress, putting the calculation of properties of heavier nuclei into a different category from that of the lighter ones. A related though distinct extension of the mathematical framework has been made by Schiff and collaborators along lines of the same general type as has been advocated without publication by Teller. According to these generalizations the meson field equations may themselves be nonlinear, and one may expect, therefore, phenomena of a different type from those arising in the older speculations on many-body forces. Conditions for saturation of nuclear forces derived for mixtures of exchange potentials may lose much of their significance if the nonlinear theory should prove to be applicable. Schiff finds in fact that one may expect strong shielding action of intervening nuclear matter on the force between two nucleons.

The subject of nuclear forces can hardly be separated from the field of nuclear physics as a whole. The discussions below are, therefore, confined to the consideration of two-body forces and to the more general features of the remaining material.

Even though the present paper is supposed to be somewhat nontechnical, it is believed that a lengthy explanation of the commonly used nomenclature in nuclear physics is superfluous. In considerations of scattering experiments the symbols s, p, d, $\cdots$ are used in the sense of indicating that the relative angular momentum in the two-body problem has values 0, 1, 2, $\cdots$, respectively. Subscripts $\frac{1}{2}$, $\frac{3}{2}$ such as $p_{\frac{1}{2}}$, $p_{\frac{3}{2}}$ denote the total angular momentum resulting from the composition of orbital and spin angular momenta. The word "phase shift" indicates the increase in the phase of a wave caused by the interaction between the particles, and the phase is defined in such a way as to contain the term kr as part of the whole expression which involves, in general, contributions from the orbital angular momentum and a progressively varying part caused by the Coulomb field. At large distances the wave is completely characterized by the phase shift. It is convenient, therefore, to make a comparison between theory and experiment by deriving phase shifts from experimental data and calculating them from different theoretical hypotheses.

EXPERIMENTAL EVIDENCE REGARDING THE TWO-BODY PROBLEM

The Deuteron

Binding Energy of the Deuteron

The word "binding energy" will be used below in such a sense as to be represented by a negative number if the nucleus is stable. The negative of the binding energy will be referred to as "binding tightness." Bell and Elliott[1] have obtained an improved value $\epsilon=-(2.230\pm0.007)$ Mev for the binding energy of the deuteron. The method was to measure the γ-ray energy for the $n(p\gamma)d$ reaction. The γ-ray energy has been measured in a β-ray spectrometer by comparing momenta of electrons ejected from a thin uranium target with those produced by the 2.615-Mev γ-ray of ThC″. Such determinations have not been made previously because sufficiently intense neutron sources have not been available to make high accuracy work feasible.

Mobley and Laubenstein[2] obtained $\epsilon=-(2.226\pm0.003)$ Mev. This information has been critically reviewed by Salpeter,[3] who also combined the Q value for $H^3(p, n)He^3$ and the mass spectroscopic mass difference between the deuteron and hydrogen molecule to obtain the independent result $\epsilon=-(2.224\pm0.006)$ Mev.

Proton Scattering of Thermal Neutrons[4]

As a result of work of Hughes and collaborators,[5] one now has good values for the coherent amplitude and for the scattering cross section of epithermal neutrons. These values are $2(3\,{}^3a/4+{}^1a/4)=(3.78\pm0.02)\times10^{-13}$ cm for the coherent amplitude and $\pi[3({}^3a)^2+({}^1a)^2]=(20.36\pm0.10)\times10^{-24}$ cm^2 for the cross section[6] of epithermal

[1] R. E. Bell and L. G. Elliott, Phys. Rev. **79**, 282 (1950).
[2] R. C. Mobley and R. A. Laubenstein, Phys. Rev. **80**, 309 (1950).
[3] E. E. Salpeter, Phys. Rev. **82**, 60 (1951).
[4] The value given below includes a slight change as in Breit and Gluckstern for *Annual Reviews* (to be published) taking account of some later values mentioned by Salpeter but not corrected for by him.
[5] Hughes, Burgy, and Ringo, Phys. Rev. **77**, 291 (1950); Phys. Rev. **79**, 227 (1950); Ringo, Burgy, and Hughes, Phys. Rev. **82**, 344 (1951).
[6] E. Melkonian, Phys. Rev. **76**, 1744 (1949).

neutrons. The convention regarding the sign of the Fermi intercepts followed here is to have $a>0$ for a small attraction so that the a and the associated small positive phase shift have the same sign. The above values give ${}^3a=-5.378(1\pm0.0038)\times10^{-13}$ cm, ${}^1a=23.69(1\pm0.0022)\times10^{-13}$ cm.

Photomagnetic Capture

The ratio of photoelectric and photomagnetic deuteron cross sections has been determined by Bishop, Halban, Shaw, and Wilson.[7] They measured the angular distribution of neutrons produced in the photodisintegration of the deuteron by the 2.51-Mev γ-rays of Ga^{72}. The theoretical relationship between the anisotropic and spherically symmetric parts of the angular distribution has then been used to determine the desired ratio, and the result is $\sigma_m/\sigma_e=0.61\pm0.04$.

Whitehouse and Graham[8] have measured the ratio of neutron absorption cross sections of boron and hydrogen, σ_B/σ_H, employing an integration method in water and in aqueous solutions of boric acid. The result is $\sigma_B/\sigma_H=(2270\pm30)$. From the value of the cross section for boron at thermal energies they obtained $\sigma_H(kT)=0.31\times10^{-24}$ cm^2/atom. This value is used in the calculations of Feshbach and Schwinger,[9] in accordance with values originally obtained by Bethe and Longmire[10] who employed the results of Fermi and Marshall[11] giving $\sigma_B=703\pm7$ barns.

Photoelectric and Photomagnetic Disintegration

The cross section for photodisintegration for low energies has been made the subject of a very thorough investigation on an international scale by Bishop, Collie, Halban, Hedgram, Siegbahn, du Toit, and Wilson.[12] They employed γ-rays of Ga^{72} and Na^{24} and measured relative cross sections at 2.51, 2.62, and 2.76 Mev.

The radiosodium source was calibrated by the $\beta-\gamma$ coincidence method developed by Dunworth and Putnam.[13] A thorough study of various corrections such as those arising from γ-ray absorption, wall effect, etc., has been made, and excellent consistency of measurements has been obtained. The values are as shown in Table I.

TABLE I. Photodisintegration cross section of the deuteron.

	Experiment	Theory
$\sigma_{2.757}\times10^{+27}$ cm^2	15.9 ±0.6	15.1 ±0.3
$\sigma_{2.618}\times10^{+27}$ cm^2	13.9 ±0.6	12.5 ±0.3
$\sigma_{2.757}/\sigma_{2.504}$	1.34 ± 0.05	1.49 ± 0.02
$\sigma_{2.618}/\sigma_{2.504}$	1.16 ± 0.04	1.24 ± 0.01
$\sigma_{2.504}\times10^{27}$ cm^2	11.9 ±0.8	10.1 ±0.3

The column marked "Theory" in Table I corresponds to calculations made by Hansson and Hulthén.[14] It is reproduced as a matter of general interest and without emphasis on the excellent though possibly fortuitous agreement between theory and experiment. In view of later developments in theoretical views it appears somewhat improbable that nuclear potentials are velocity independent, and the pseudoscalar theory with a repulsive core came into being considerably later than the work just quoted. On the other hand, the photoelectric disintegration is not too sensitive to some changes in nuclear force parameters.

Low Energy Proton-Proton Scattering

Very accurate measurements have been made at the University of Wisconsin within the last year by the group working with Professor R. G. Herb. In the same connection one should mention the work of Rouvina,[15] Heydenburg and Little,[16] Yntema and White.[17] Other and generally older work is referred to in connection with graphical presentation of the data. The new Wisconsin material is believed to be of a much higher accuracy than any other measurements, and it is believed that it contains evidence of scattering other than pure s scattering. This work may be considered as a natural development of the earlier investigation of Herb, Kerst,

[7] Bishop, Halban, Shaw, and Wilson, Phys. Rev. **81**, 219 (1951).
[8] W. S. Whitehouse and G. A. R. Graham, Can. J. Research **A25**, 261 (1947).
[9] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951).
[10] H. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950).
[11] E. Fermi and L. Marshall, Phys. Rev. **72**, 193 (1947).
[12] Bishop, Collie, Halban, Hedgram, Siegbahn, du Toit, and Wilson, Phys. Rev. **80**, 211 (1950).
[13] J. R. Dunworth, Rev. Sci. Instr. **11**, 167 (1940); J. L. Putnam, Brit. J. Radiol. **23**, 265 (1950).
[14] I. F. E. Hansson and L. Hulthén, Phys. Rev. **76**, 1163 (1949).
[15] J. Rouvina, Phys. Rev. **81**, 593 (1951).
[16] N. P. Heydenburg and J. L. Little, private communication.
[17] J. L. Yntema and M. G. White, Technical Report, NYO 3478, Princeton University, May 15, 1952.

Parkinson, and Plain[18] which has served as a model of accuracy and precision in the field for many years. Painstaking precautions have been taken by Worthington, Findley, and McGruer to eliminate errors which might arise from contamination of hydrogen by other gases. In the older Wisconsin measurements care has been taken to avoid false counts which might result from scattering by nuclei of oxygen, nitrogen, and other impurities. Since the Rutherford formula gives a scattering yield proportional to Z^2, serious errors may be caused by relatively slight contaminations. Such errors matter if it is desired to detect deviations from a pure s wave scattering anomaly. The angular dependence of scattering from oxygen is, in fact, quite different from that of hydrogen. The older method consisted in visual monitoring of scattering pulses by means of oscilloscope at scattering angles where the scattering from hydrogen was sure not to interfere with much larger pulses arising from the impurity. In the new measurements a special contamination counter was used during the regular runs. It viewed the incident beam at an angle of about 90° with respect to the direction of incident protons. The counter window was thick enough to stop stray scattering. The same counter was used with hydrogen in the chamber. It was assumed that at a small scattering angle (20°) the contamination scattering varied with energy according to Rutherford's formula. The correction for scattering from contamination was then calculated under the assumption that the composition of the impurities is the same with and without hydrogen. The Wisconsin group is well aware of the fact that this assumption is not completely justified. Indeed they traced approximately 75 percent of the contamination counts to contaminants carried into the chamber by the hydrogen. However, the corrections were usually less than 0.1 percent for scattering angles of 25° or larger. The probable error of the correction is considered to be about 30 percent of the correction itself, and the probable error of the yield for angles greater than 25° is accordingly about 0.03 percent. The uncertainty of 30 percent in the contamination correction has been arrived at from the observation that during regular scattering runs there occur occasional jumps in the contamination yield which give rise to changes in counts at different angles; these counts do not reproduce the angular variation expected from runs in the absence of hydrogen to better than 30 percent.

The correction increases rapidly at small angles and it is, therefore, important in affecting conclusions regarding the presence of phase shifts for angular momenta $L>0$. It is about -0.7 percent at 15° for a bombarding energy of 1.855 Mev. Work is in progress on improvements in the knowledge of the correction from studies of the scattering of protons by oxygen, nitrogen, and carbon.

The high precision of the new work has been made possible through the construction of an electrostatic analyzer capable of defining the energy of the incident beam to better than 0.1 percent. This analyzer has been designed by Herb[19] and the voltage scale was determined by Herb, Snowden, and Sala.[20] The reference standard is the threshold of the $Li^7(p,n)Be^7$ reaction which according to the above work is 1.882 ± 0.02 Mev. A correction for energy loss in the hydrogen between the entrance to the scattering chamber and the scattering volume had to be applied. Improvements have been made in the measurement of charge carried in the primary beam, and in corrections for slit edge scattering in the analyzer, in the collimator, and in the counter assembly. The availability of better equipment such as polystyrene condensers and general advances in electronic techniques have been of obvious importance in the possibility of refining the older work of HKPP which has been considered as a model of precision for some time. The whole development has been carefully planned since 1948 and the period of practice measurements and the reduction of data with an application of the necessary corrections has taken several years.

A related attempt at attaining high precision has been made by Yntema and White[17] at Princeton. Their scattering chamber is also of a much improved design in comparison with those used before the war, and the precautions

[18] Herb, Kerst, Parkinson, and Plain, Phys. Rev. **55**, 998 (1939).

[19] Warren, Powell, and Herb, Rev. Sci. Instr. **18**, 559 (1947).

[20] Herb, Snowden, and Sala, Phys. Rev. **75**, 246 (1949).

TABLE II. Proton-proton scattering at 18.2 Mev according to J. L. Yntema and M. G. White.

Center-of-mass angle	Center-of-mass cross section in millibarns	Relative probable error (percent)	Absolute probable error (percent)
90°	27.32	±0.4	0.9
80°	27.29	±0.5	
70°	27.47	±0.5	
60°	27.42	±0.6	
50°	27.27	±0.7	
40°	26.50	±0.8	
36°	25.98	±1.0	
30°	25.00	±1.0	

taken are similar to those at Wisconsin. The observations of Yntema and White were made at a bombarding energy of 18.2 Mev. In view of the fact that this energy is higher than that of the Madison work, one might hope that definite indications of deviation from a pure s wave anomaly can be obtained with a smaller precision in the measurements. The results of the Princeton workers are reproduced in Table II.

Effective Potentials Representing Low Energy Proton-Proton Scattering

Since the pioneer experiments of Tuve, Heydenburg, and Hafstad[21] it has been customary to interpret observations in terms of effective potential energy curves. It has been realized by most of the workers in the field that such an interpretation has a provisional value at most and that the more proper way would consist in finding a theoretical view containing either an explicit connection with a field theory or some other fundamental principle. Nevertheless, the potential energy curves form a useful working hypothesis and are widely used in relating scattering data to other nuclear phenomena. The more basic aspects of the problem are touched on elsewhere in the present report, and the immediate discussion, therefore, will be in terms of effective potentials.

The potentials used in nuclear physics are of a more general type than those employed in ordinary atomic physics. Through the work of Heisenberg,[22] Majorana,[23] Wigner,[24] and others it has become clear that it would be more satisfactory to generalize the concept of potential energy, at least to the degree commonly referred to as "exchange forces." The origin of the idea of exchange forces has been in an analogy with the behavior of atoms in molecules. If one wishes to refer to the atoms as units, abandoning in such a phenomenologic description the complete representation of all of the electrostatic effects taking place between the individual electrons and nuclei, the usual Heitler-London theory gives effective potentials between atoms described as a combination of "ordinary" and of "exchange" terms in the potential energy. The "exchange" terms have to do with the possibility of electrons exchanging positions between the atoms. Guided by this analogy Heisenberg[23] postulated the existence of similar forces between nucleons. At the time there had been no experimental evidence concerning mesons and his viewpoint was of a more general character. The idea has been modified later by Yukawa's[25] view according to which the "electron" responsible for nuclear forces should be considered as a heavy electron and the latter has now become a meson. The correctness of elaborations on Yukawa is perhaps not of immediate concern, but they are usually of the more general type just mentioned. Exchange potentials are a special case of what are referred to as velocity dependent potentials, in Wheeler's terminology.[26] The most general velocity dependent potentials include the possibility of so many parameters that their employment has proved so far to be impractical and, therefore, their specializations are usually considered. For exchange potentials it is customary to deal with the standard types referred to as the Majorana, Heisenberg, spin exchange (Bartlett)[27] potentials. Principally through Wigner's work one has a rather complete understanding of how the collapse of heavy nuclei can be prevented as a result of the use of such exchange forces. When two nucleons collide under the influence of a hypothesized exchange potential they may be considered as interacting with each other by means of ordinary—non-exchange—forces provided the potential energy is made to assume different values depending on

[21] Heydenburg, Hafstad, and Tuve, Phys. Rev. **56**, 1078 (1939).
[22] W. Heisenberg, Z. Physik **77**, 1 (1932).
[23] E. Majorana, Z. Physik **82**, 137 (1933).
[24] E. Wigner, Proc. Nat. Acad. Sci. **22**, 662 (1936).
[25] H. Yukawa, Proc. Phys.-Math. Soc. Japan **17**, 48 (1935).
[26] J. A. Wheeler, Phys. Rev. **50**, 643 (1936).
[27] J. H. Bartlett, Phys. Rev. **49**, 102 (1936).

the relative angular momenta and the relative spin orientations of the colliding particles. Thus, for example, the Majorana exchange force, when postulated to give effectively an attraction in s states, will result in a repulsion in p states, an attraction in d states, etc. This alternation of attractions and repulsions gives in fact a qualitative explanation of the saturation of nuclear forces. On account of the effects of relative angular momenta and of relative spin orientations on the character of the interaction, it is not to be expected that the potential energy curves derived on the basis of evidence originating in s phase shifts is in any specially simple relation to that in p or d states. Some probable upper bounds for the interactions, consistent with saturation requirements and with other nuclear data, have been derived through the efforts of Wigner,[24] Feenberg[28] and others,[29] but these limits have never been considered too literally because of the possibility of the existence of many-body forces.[30] Most of the work on the saturation of nuclear forces has been done without the inclusion of effects of tensor forces, and the arbitrary assumption of the identity of shape and range of the interacting potentials for the four standard parts has usually been made. Conclusions regarding upper bounds for the magnitude of the p interaction based on this assumption are questionable. It would be much safer, therefore, to rely on theory as little as possible and to determine the interactions in states of angular momenta greater than zero from experimental data as directly as possible. Unfortunately this plan is difficult to carry out.

It has been fortunate for the early development of the subject that the experimental material has indicated the dominance of the s wave scattering anomaly. The determination of the s wave phase shift (K_0) from the experimental data is subject to precautions and reservations regarding the possible presence of phase shifts corresponding to higher values of L. There has been a considerable interest, therefore, in the detection of effects that could be attributed to such K_L, both on account of the intrinsic interest which they would have in revealing the character of the phenomena as well as on account of the effect which an undetected K_2 could have on the magnitude of the effective potential for the s state derived from data. The analysis of the scattering of protons by protons is especially simple on account of the absence of the ^3S state. The interference of the scattering anomaly with the Coulomb wave fortunately enables the determination of the sign of K_0.

In the discussion of effective potentials it has become customary to use certain conventions. One speaks of a shape of a potential energy curve in the sense that a potential energy represented by the formula $Af(r/a)$ is said to have the same "shape" as long as the function f is unchanged. One speaks of A as the depth parameter and of a as the range parameter. It is generally customary to take a potential such as that just referred to, to be acting in addition to the Coulomb potential e^2/r, which is present between the protons. The usual convention is to suppose that e^2/r is a valid representation of the effect of the Coulomb potential down to $r=0$. It is thus assumed for the sake of simplicity and as a matter of eliminating too much flexibility in the interpretation that the electrical properties of the protons are such as though they were point charges. These assumptions have never been meant to be more than temporary expedients for there must clearly be limitations on their validity, and one would expect that the assumption concerning the Coulomb force is not true for distances of the order of $\hbar/Mc$. Decided reservations could also be made regarding the probability that they apply exactly for reasonably small fractions of e^2/mc^2. Corrections for changes in the assumptions, regarding the region within which a literal interpretation of e^2/r applies, can easily be made by perturbation calculations, but there has not been much reason for selecting one rather than another distance at which e^2/r should be modified.

The shapes of potential energy curves that have become customary are the Gauss error potential Ae^{-ar^2}, the exponential potential $Be^{-r/b}$, and the meson potential $Ce^{-r/a}/(r/a)$. The reason for the popularity of the first two is their convenience in making calculations. The Gauss error

[28] E. Feenberg, Phys. Rev. **52**, 667 (1937).
[29] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936); N. Kemmer, Nature **140**, 192 (1937); H. Volz, Z. Physik **105**, 537 (1937); G. Breit and E. Wigner, Phys. Rev. **53**, 998 (1938).
[30] H. Primakoff and T. Holstein, Phys. Rev. **55**, 1218 (1939).

potential has been very popular in the treatment of binding energies of light nuclei on account of the closed expressions for the integrals arising in such calculations. The meson potential appeared to be a probable form on the basis of Yukawa's theory. One of the most common potentials employed is the so-called square well. The potential energy in this case is assumed to have a constant value for $r<b$, where b is the range parameter, while for $r>b$ the potential is taken to be zero. For the square well, if one assigns to b the value e^2/mc^2, one fits the data at the lowest energies by employing a depth D of ~ 11 Mev for the ^1S state.

The s wave interaction between two protons is almost large enough to give resonance at zero energy. This fact may be accidental but it may do no harm to mention it, there being as yet no complete theory of nuclear forces.

The relatively large size of the attraction between the protons in the s state is responsible for a simplification in the relationships of effective potential energy curves to the data. As the energy of the bombarding protons is varied, the effective kinetic energy inside the potential well changes also. Only half of the bombarding energy, however, contributes to the kinetic energy inside the well, because the other half is consumed as the kinetic energy of motion of the center of mass of the two particles. As a result a change of 5 Mev in the bombarding energy produces a change of 2.5 Mev in the kinetic energy inside the potential well, and this change is relatively small in comparison with the kinetic energy caused by the nuclear attraction. For this reason the shape of the s wave function inside the well changes relatively little in the low energy range $0<E<5$ Mev, and the effect of changes in E can be taken account of in first approximation by assuming it to vary linearly with E. The important quantity is the so-called homogeneous logarithmic derivative, $Y=rd\mathfrak{F}/\mathfrak{F}dr$, of $r\times$the radial wave function $\mathfrak{F}$. This quantity matters because it determines the ratio of slope to value for the wave function outside the well and hence the phase shift K_0. The possibility of taking account of the changes in $Y=[rd\mathfrak{F}_i/\mathfrak{F}_i dr]_{r=b}$ by a linear approximation simplifies the relationship between the phase shift and energy. It is found, in fact, that

$$\left[\frac{rd\mathfrak{F}_i}{\mathfrak{F}_i dr}\right]_{r=b} = \left\{\left(\frac{r}{a}\right)\frac{X_0+[f+2\ln(2r/a)]\Phi_0^*}{\Psi_0+(r/a)[f+2\ln(2r/a)]\Phi_0}\right\}_{r=b}, \quad (1)$$

where Φ_0, Φ_0^*, Ψ_0, X_0 are power series in the energy which arise in the evaluation of Coulomb functions and their derivatives. The quantity a is the Bohr collision length

$$a=\hbar^2/\mu e^2 \quad (1.1)$$

and

$$f=(C_0^2/\eta)\cot K_0-2\ln\eta+q_0/\eta, \quad (1.2)$$

where

$$q_0/2\eta=2\gamma-1+\text{R.P.}[\Gamma'(i\eta)/\Gamma(i\eta)], \quad \gamma=0.5772\cdots=\text{Euler's constant}, \quad (1.3)$$

so that

$$q_0/2\eta=-1/1+\eta^2+\Sigma_2^\infty\eta^2/[s(s^2+\eta^2)] \quad (1.3')$$

and

$$C_0^2/\eta=2\pi/(e^{2\pi\eta}-1), \quad (1.4)$$

while

$$\eta=e^2/\hbar v, \quad 1/\eta^2=40.01E_{\text{Mev}}. \quad (1.5)$$

The series for Φ_0, Φ_0^*, X_0, Ψ_0 may be found in the original papers.[31] For purposes of orientation a few terms are listed below:

$$\begin{aligned} X_0&=2-(4+\eta^{-2})(r/a)-4(r/a)^2+\cdots,\\ \Phi_0&=1+(r/a)+[(1/3)-(1/6\eta^2)](r/a)^2+\cdots,\\ \Phi_0^*&=1+2(r/a)+[1-(1/2\eta^2)](r/a)^2+\cdots,\\ \Psi_0&=1-[3+(1/2\eta^2)](r/a)^2+\cdots. \end{aligned} \quad (1.6)$$

The arrangement of terms in Eq. (1) is such as to remove the logarithmic term in energy and distance from everything but the combination $f+2\ln(2r/a)$. Since the solution of Eq. (1) for f is obviously a power series in the energy and since for a potential of zero range the limiting form of Eq. (1) expresses f entirely in terms of constants, one has

$$f=\text{const}(b=0). \quad (1.7)$$

In this case Eq. (7.9) of Breit, Condon, and Present[31] is applicable; and it becomes, on clear-

[31] Yost, Wheeler, and Breit, Phys. Rev. 49, 174 (1936); Breit, Condon, and Present, Phys. Rev. 50, 825 (1936).

ing fractions,

$$f=f_{,}'$$

where f' is the value of f evaluated at a standard energy for which K_0 has a preassigned value. The discussion just referred to applies to potential energy curves of any shape. It gives[32]

$$[f+2\ln(2(r/a))-(X_0-(a/r)Y\Psi_0)/(Y\Phi_0-\Phi_0^*)]_{r=b}=0, \quad (2)$$

where

$$Y=rd\mathfrak{F}/\mathfrak{F}dr. \quad (2')$$

As long as Y may be expanded in powers of the energy, this relation gives for f an expression which is the ratio of two power series. The quantities X_0, Ψ_0, Φ_0, Φ_0^* are closely related to hypergeometric functions; and the series converge for all energies. The convenience of the function f is that the plot of it against E is nearly linear at low energies. It is convenient, therefore, to expand it as

$$f=f^{(0)}+f^{(1)}E_{\text{Mev}}+f^{(2)}E_{\text{Mev}}{}^2+\cdots.$$

It may be shown that[33]

$$[f-f^{(0)}-(f'-f'^{(0)})]/[a(k-k_0)]=\Sigma_0{}^{\infty}\int_0^b[v_s'^2-v_s^2]dr+\Sigma_0{}^{\infty}\int_0^b[v_s'v_{s+1}'-v_sv_{s+1}]dr, \quad (3)$$

where

$$u=\Sigma_0{}^{\infty}v_n, \quad v_0=u_0, \quad u'=\Sigma_0{}^{\infty}v_n', \quad v_0'=u_0', \quad (3.1)$$

it being understood that the successive terms in the expansion of u in Eq. (3.1) are the same as in

$$u=\Sigma_0{}^{\infty}(\kappa-\kappa_0)\ {}^nu_n, \quad (3.2)$$

where u satisfies

$$(d^2u/dr^2)+[\kappa-\kappa_0+\cdots]u=0, \quad (3.3)$$

with

$$\mathfrak{IC}-\mathfrak{IC}_0=k^2-k_0{}^2. \quad (3.4)$$

Here $'$ indicate quantities obtained for a comparison potential. It is convenient to choose a potential of zero range in order to define f' thus insuring the energy independence of f'.

The linearity of f depends essentially on the fact that Y is not very sensitive to E, so that a linear formula for Y in Eq. (2) gives a good approximation to f. The lack of sensitivity of Y to E is the result of the relatively large absolute value of the potential energy inside the potential well.

The function f was first introduced by Breit, Condon, and Present.[31] Schwinger[34] is responsible for its treatment by means of the variational approach. Peierls and Preston[35] have pointed out how the same result can be obtained by means of well-known relations between $\partial Y/\partial E$ and integrals such as Eq. (3). Bethe,[36] going as far as $\partial f/\partial E$, and Chew and Goldberger[37] have made numerous applications and extensions which have been especially valuable in the comparison of $p-p$ and $p-n$ material. Blatt and Jackson[38] have applied the variational approach of Schwinger to the analysis of $p-p$ data in their comprehensive article in the *Reviews of Modern Physics*. Breit[33] has shown how the results of the variational method follow in a simple manner from the definition of f and how they can be generalized to include expansions in terms of parameters describing the potential energy. Unfortunately various authors employ different definitions of this function. The relationships to the original function f are

$$\mathbf{K}=\tfrac{1}{2}f+1-2\gamma=\tfrac{1}{2}f-0.15443,$$
$$f_{\text{Bethe}}=f+2-2\gamma=f+0.84557.$$

The function f assumes an especially simple form if the charge e is made to approach zero in the formulas. In this limit

$$f/a=k\cot K_0.$$

A newer analysis of experimental material has been made by Yovits, Smith, Hull, Bengston, and Breit.[39] The fits of the function f have been made in this case employing various criteria for weighting of experimental data. One of these is

[32] G. Breit and W. Bouricius, Phys. Rev. **75**, 1029 (1949).
[33] G. Breit, Revs. Modern Phys. **23**, 238 (1951).
[34] J. Schwinger, Phys. Rev. **72**, 742 (1947).
[35] R. E. Peierls and M. A. Preston, Phys. Rev. **72**, 250 (1947).
[36] H. A. Bethe, Phys. Rev. **76**, 38 (1949).
[37] G. F. Chew and M. L. Goldberger, Phys. Rev. **75**, 1637 (1949).
[38] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).
[39] Yovits, Smith, Hull, Bengston, and Breit, Phys. Rev. **85**, 540 (1952); referred to as YSHBB.

TABLE III. *f*-Function fits for experimental proton-proton scattering data.

Type of potential well	Type of fit	$f(0)$	$f'(0)$	$f''(0)$
Meson	Quadratic[a]	7.788 ± 0.009	0.952 ± 0.005	−0.00948
Exponential	Quadratic[b]	7.810	0.922	−0.0015
Square well	Quadratic[b]	7.825	0.899	0.0048
None	Quadratic	7.785	0.956	−0.0104
None	Linear[c]	7.811	0.917	0

[a] Mean of internal and mixed criteria used for relative weight assignment.
[b] These fits have not been made as accurately as the fit for the meson well.
[c] In this and in the preceding fit an ordinary least squares fit is made.

based primarily on the internal consistency of the data, the possibility of fitting the data by means of a suitably adjusted f function being taken as the criterion of goodness; the other was based on the way in which the data of a given set of workers can be made to agree with data from other laboratories. It was felt that the presence of systematic errors can hardly be estimated without employment of some definite criteria. Results obtained by means of the criteria of internal consistency are referred to as $\mathcal{I}$ and

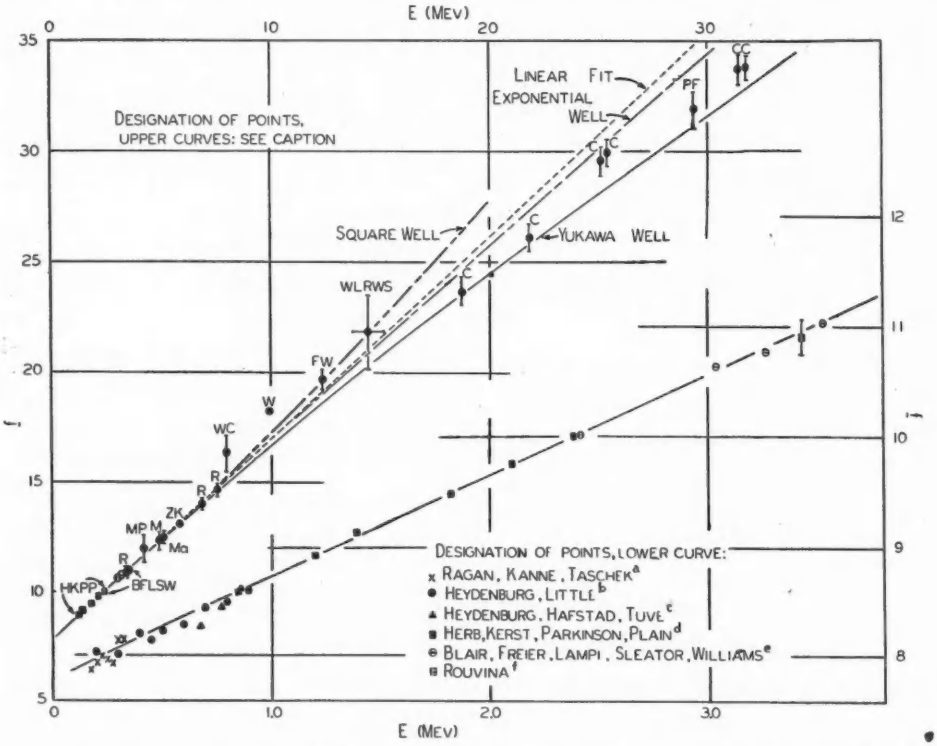

FIG. 1. Same as Fig. 1 of YSHBB (reference 40). *Lower curve:* Plot of f *vs* E for best fits with experimental points as designated, using scales on right and lower margins. *Upper curves:* Plot of f *vs* E for best fits as labeled with experimental points having the following designations: HKPP—Herb, Kerst, Parkinson, and Plain;[d] BFLSW—Blair, Freier, Lampi, Sleator, and Williams;[e] R—Rouvina;[f] MP—May and Powell, Proc. Roy. Soc. (London) **A190**, 170 (1947); M—Meagher, Phys. Rev. **78**, 667 (1950); Ma—Mather, Phys. Rev. **82**, 133 (1951); ZK—Zimmerman and Kreuger, Phys. Rev. **83**, 218 (1950); WC—Wilson and Creutz, Phys. Rev. **71**, 339 (1947); W—Wilson, Phys. Rev. **71**, 384 (1947); FW—Faris and Wright, Phys. Rev. **79**, 577 (1950); WLRWS—Wilson, Lofgren, Richardson, Wright, and Shankland, Phys. Rev. **71**, 560, (1947); C—Cork, Phys. Rev. **80**, 321 (1950); PF—Panofsky and Fillmore, Phys. Rev. **79**, 57 (1950). The scales on the left and upper margins are used with the upper curves.

[a] Ragan, Kanne, and Taschek, Phys. Rev. **60**, 628 (1941).
[b] N. P. Heydenburg and J. L. Little, private communication to YSHBB.
[c] Heydenburg, Hafstad, and Tuve, Phys. Rev. **56**, 1078 (1939).
[d] Herb, Kerst, Parkinson, and Plain, Phys. Rev. **55**, 998 (1939).
[e] Blair, Freier, Lampi, Sleator, and Williams, Phys. Rev. **74**, 553 (1948).
[f] J. Rouvina, Phys. Rev. **81**, 593 (1951).

those from a combination of internal and external consistencies by the letters $\mathcal{SE}$. Their results are given in Table III. The fits obtained above are represented in Fig. 1, while a graphical fit to data for the Yukawa potential is represented in Fig. 2. The resulting Yukawa parameters are: $\mathcal{C}=(93\pm1)mc^2$, $a=(0.412\pm0.002)\times e^2/mc^2$.

It will be noted that the mass of the meson resulting from these fits is ~330 m. This value of the mass has remained rather stable to changes in experimental values.[40] The mass of the meson at the time of the first comparisons was very uncertain. It turned out later that most of the mesons measured in the early period were not of the type that can be held responsible for nuclear forces, and it was not until μ-mesons became recognized to be distinct from π-mesons that comparisons became meaningful. The mass of the π-meson is lower (~280) than the effective mass of the meson derived from $p-p$ scattering. Only tentative explanations of this discrepancy are available. Breit and Bouricius[33] have pointed out in this connection that it may be unreasonable to interpret $p-p$ scattering too literally by means of potential energy curves and that for this reason one could perhaps take the viewpoint of a temporary formation of a compound state when the two protons collide. If one then supposes that a state is of such character that its dissociation is determined primarily by the motion of mesons then one can expect that the kinetic energy of bombarding protons has much less effect on the logarithmic derivative Y than one obtains from calculations based on potential energy curves. The simplest possible hypothesis was tried, namely that the logarithmic derivative is independent of the bombarding energy. It then turned out possible to account for the low energy experimental material about as well as one does by means of potential energy curves. The fits are reproduced in Fig. 3 and Fig. 4. According to this decidedly oversimplified picture, the two protons could be considered as forming a compound state whenever the distance between them is smaller than 0.47 e^2/mc^2 and for larger distances the protons are considered as not exerting any force on each other, except for the Coulomb force. The fact that such a fit can be obtained may be accidental, and its explanation may perhaps be found in the relations discussed by Breit and Yovits[41] in their Eqs. (5), (5.1), and (5.2). On the other hand it appears interesting that such an extreme point of view which differs altogether from the adiabatic approximation underlying the meson theoretic explanation of the nuclear potentials can be successful. The situation suggests that agreements between potential energy curves and experimental data do not necessarily indicate that the potential energy curves are a physically correct representation of the phenomenon. The principal difference between the two views consists in the treatment of kinetic energy of the bombarding protons. For the potential energy curve type of explanation, one supposes that all of the kinetic energy available for relative motion between the protons when they are free remains available for their relative motion when they are within the potential well. On the other hand, the boundary condition point of view supposes that in the region where the two protons interact it becomes practically meaningless to speak of a relative motion between the protons without simultaneously considering the motions of the mesons. The kinetic energy of relative motion goes in this case primarily into the meson energy,

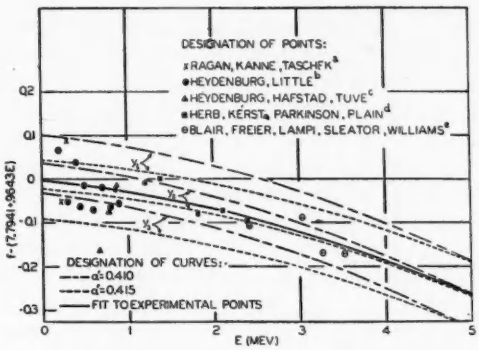

FIG. 2. Same as Fig. 3 of YSHBB (reference 40). Theoretical values of $f-(7.7941+0.9643E)$ for $a'=0.41$ and 0.415 with y_1, y_2, y_3 having values 0.2514372, 0.2511965 and 0.2509653, respectively, plotted against E, together with least squares fit to data and experimental points as designated. The quantity $y=1/(\mathcal{C}'a'^2)^{\frac{1}{3}}$. The reference letters are the same as for Fig. 1.

[40] Hoisington, Share, and Breit, Phys. Rev. **56**, 884 (1939); G. Breit and R. D. Hatcher, Phys. Rev. **78**, 110 (1950); J. D. Jackson, thesis, MIT, 1949, p. 93 of ONR report.

[41] G. Breit and M. C. Yovits, Phys. Rev. **81**, 416 (1951).

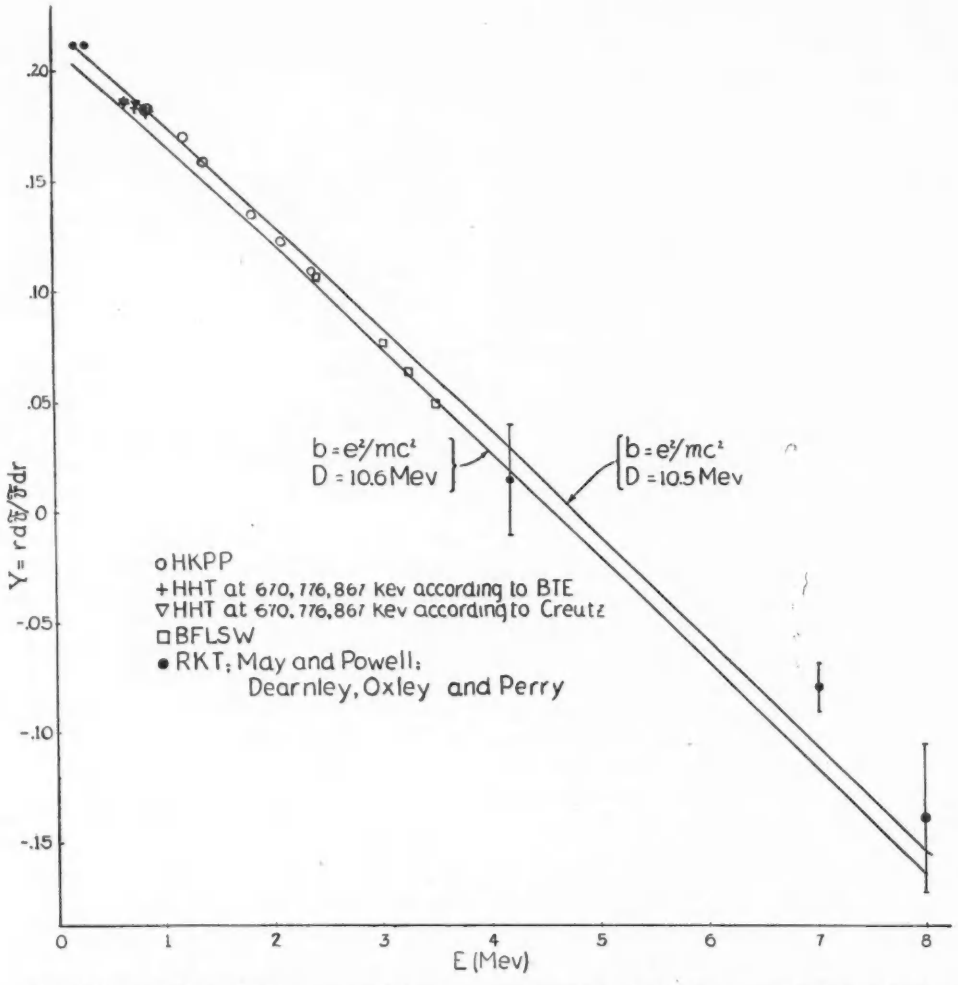

FIG. 3 Same as Fig. 1 of BB (reference 33). Homogeneous logarithmic derivative at $r = e^2/mc^2$ plotted against energy. Designation of experimental points is as shown with references the same as in Fig. 1. The analyses of both BTE (reference 43) and Creutz, Phys. Rev. 56, 893 (1939) are used for the HHT points. The comparison curves correspond to assumed square well potentials of radius e^2/mc^2 and depths 10.5 and 10.6 Mev with no Coulomb energy within the well.

and the dissociation of the protons takes place independently of the bombarding energy.

A related consideration is carried out by Breit and Yovits[41] who have considered the possibility of the formation of an isobaric state at an energy comparable with the mass of the π-meson. They find that it is then not necessary to make use of a formal meson theory in order to account for nuclear forces. The existence of the isobaric state can account for a large portion of the interaction. They also find that the isobaric state results effectively in a velocity dependence of the potential. The existence of the velocity dependence follows directly from the postulated isobaric state and simple considerations of second-order Schroedinger perturbation theory. The order of magnitude of velocity dependence is such that, if there were a potential energy curve attributable to the more standard Yukawa-type consideration, the effective attraction be-

tween the particles would have to be considered as increasing whenever the bombarding energy of the protons is increased. A simple consideration shows then that the range of the force would be judged to be somewhat shorter. The sign of this correction for velocity dependence is such as though the meson mass were larger than the actual one and is in apparent agreement with observations on the mass of the π-meson and on the slope of the f curve. The possibility of errors in the determination of the range parameter of a nuclear potential arising from a velocity dependence has been previously discussed by Breit, Thaxton, and Eisenbud.[42]

The pseudoscalar meson theory of nuclear forces shows evidence of explaining the difference between the apparent and observed mass of the π-meson as a fourth-order effect in the meson-nucleon interaction. This matter will be briefly reviewed in a later section in connection with the work of Lèvy and of Lepore. But it may be mentioned here that the boundary condition used by Breit and Bouricius[32] may be considered as the limit of the condition of an infinitely high repulsive core, surrounded by a very narrow attractive region within which the attraction is made strong enough to make the logarithmic derivative finite on its outer boundary. By decreasing the width of the attractive region to zero one makes the potential well depth infinite, and the logarithmic derivative on the outer boundary becomes energy independent as in the work of Breit and Bouricius. The primary motivation of the work just quoted has a different appearance from that of the developments of the pseudoscalar theory. On the other hand, in both views the large meson density at short distances is essential, and a closer examination of the velocity dependence on the pseudoscalar theory will have to involve a consideration of the manner in which the mesons participate in the sharing of the kinetic energy of the colliding particles.

HIGH ENERGY SCATTERING[43]

Scattering Experiments

The striking features of high energy proton-proton scattering are: (a) approximate constancy of the observed cross section per unit solid angle in the center-of-mass system as the scattering angle is varied; (b) approximate constancy of the cross section as the energy is varied; (c) comparatively large value of σ which is too large to

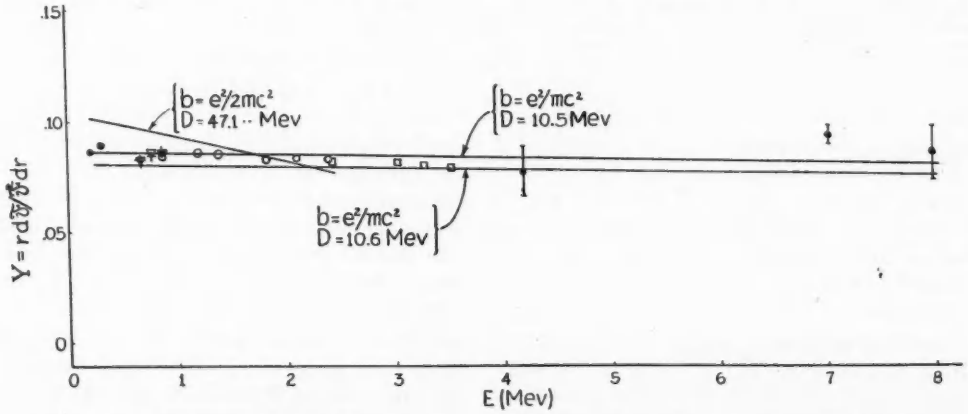

FIG. 4. Same as Fig. 2 of BB (reference 33). Values of Y at $r=e^2/2mc^2$, obtained by continuation of the Coulomb function $\mathfrak{F}=F\cos K_0+G\sin K_0$ to that distance. The two nearly horizontal lines correspond to phase shifts K_0 computed for the square wells used in Fig. 3. The oblique line corresponds to a square well with range $=e^2/2mc^2$, depth $=47.14$ Mev. The experimental points are designated as in Fig. 3.

[42] Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1018 (1939).

[43] Some of the material in this section overlaps portions of "Advances in Nucleon-Nucleon Scattering Experiments and Their Theoretical Consequences" by G. Breit and R. L. Gluckstern, to be published in the *Annual Reviews*. This reference contains a more complete account of experimental information and a more technical presentation of some parts of the theory. The authors would like to express their indebtedness to Dr. R. L. Gluckstern for permission to use parts of the paper quoted.

be explained by ^1S scattering. The situation is reviewed in detail in a paper by Chamberlain, Segrè, and Wiegand[44] (hereafter referred to as CSW) which gives an up-to-date account of experimental results on $p-p$ scattering, the more

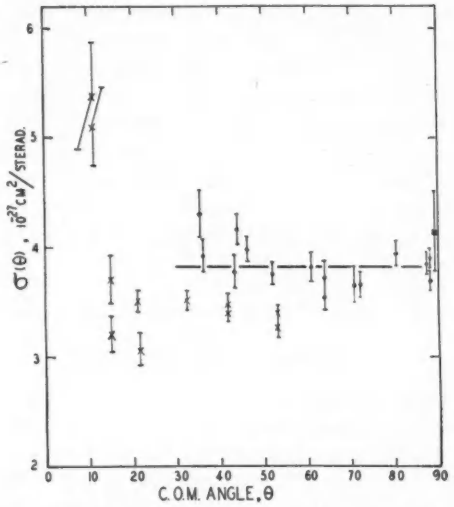

FIG. 5. Redrawn from Tables I and II of CSW (reference 44) to correspond to their Fig. 10. Differential cross section in center-of-mass system for $p-p$ scattering at 345 Mev obtained by CSW. Standard deviations derived from counting statistics are shown as uncertainties. The scattering angle is shown in degrees. Crosses and square at ~90°: liquid hydrogen target, single counter method. Circles: CH_2 target, coincidence method.

relevant $n-p$ scattering data and discusses comparison of experiment with theory. The source of protons is the external beam of the Berkeley 184-inch cyclotron. The targets were of polyethylene or liquid hydrogen, the latter being used for small angles. Counting was done with stilbene crystals singly (for small angles) and in coincidence.

The results support earlier conclusions regarding the approximate lack of dependence of the scattering cross section in the center-of-mass system on the scattering angle and also regarding the approximate independence of the cross section on energy. The results at 345-Mev bombarding energy are shown in Fig. 5. There are two sets of points in this figure, the circles referring to measurements with polyethylene targets by the coincidence method, the crosses to the liquid hydrogen target by the single counter method. The latter is presumably the less accurate except at the smaller angles. It is impractical therefore to use one set of measurements as an accurate check on the other. At an angle of ~10° the single counts method gave a marked and unexplained rise in σ_s. The geometry being less favorable for accuracy at such angles than at larger ones, the approximate agreement of the two methods for $30°<\theta<50°$ is hardly relevant as a criterion in forming an opinion regarding the rise. A larger-than-estimated angular spread of the primary beam and a larger than estimated component of low energy protons would both tend to produce too large scattering at small angles.

In evaluating experiments in the 100–350 Mev, range it is perhaps advisable to exercise more caution than in the "low energy" range from 0.2–15 Mev. The instrumentation is relatively new and experience in its use is still limited. The large size of the equipment makes changes difficult. It is common experience that sources of error are difficult to locate unless the experimenter is free to change the arrangement and the dimensions of component parts of his apparatus and to check the resulting changes in his results against the theory of the experiment. Such changes require more thought and planning in the high energy region.

The correction for background caused by the scattering from carbon in polyethylene has been carefully considered by Segrè *et al.*[44] A superficial study of the work might leave one in doubt regarding the adequacy of these corrections. The special circumstance is that Segrè *et al.*[44] have found that protons are knocked out of Li^7 at 345 Mev, mostly at 85° with respect to the incident direction as though they were free. There are similar knockout protons from carbon.[45] If the angle between the scattered and knockout protons were exactly the same as for free protons there would be doubt regarding the adequacy of the background corrections. In this case coincidences would not distinguish between the protons scattered from hydrogen and those from

[44] Chamberlain, Segrè, and Wiegand, Phys. Rev. **83**, 923 (1951).

[45] E. Segrè, "Proc. Int. Conf. Nuc. Phys. and Phys. of Fund. Part." p. 87 (1951), compare, especially p. 94, in response to a question by G. Breit.

carbon. The authors have ascertained,[46] however, that the knockout protons are distributed in a cone with a half-value breadth of ~22° from the mean direction. This width they ascribe to the momentum distribution of protons in the carbon nucleus. The effect of the width is to reduce the number of coincidence counts caused by a scattered proton and its partner knockout proton to a negligible fraction of the number of coincidence counts from hydrogen. The correction for background coincidence counts from carbon is made, therefore, on the basis of these counts being accidental. They arise mainly from accidental coincidences of protons coming from different carbon nuclei. Their number has been ascertained by substituting a carbon target for polyethylene and subtracting 0.6 of the carbon target coincidences from the polyethylene coincidences. The number 0.6 is arrived at employing an experimentally observed ratio of 5:1 of single counts from carbon and hydrogen and the ratio of solid angles subtended by the two crystals. The reasoning is presented in detail in the section on "Targets" of the original paper.[44] The procedure was tested by comparing results at different beam intensities.

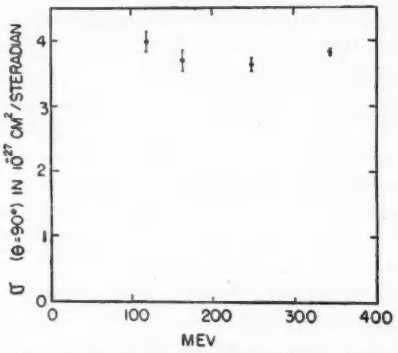

FIG. 6. Same as Fig. 11 of CSW (reference 44). Differential $p-p$ scattering cross section at $\theta=90°$ as a function of incident proton energy. Units: σ in 10^{-27} cm²/sterad, E in Mev. Uncertainties are indicated in terms of standard deviations derived from counting statistics.

Observations by other investigators made in other laboratories give approximately the same results as those obtained in the pioneer experiments of the Berkeley group. The numbers quoted above show, however, some of the inherent difficulties of the problem. It is encouraging to know, therefore, that new experiments are being planned at Berkeley and elsewhere; it is important to make sure that unavoidable experimental inaccuracies have not masked nonuniformities in angular distribution.

Taking the results at face value, one sees that the single counter measurements by themselves show a drop from about 4.1 mb/sterad at $\theta=90°$ to perhaps 3.5 mb/sterad at 20° and again a rise to about 5 mb/sterad at 11.3°. If one wishes to, one can read some angular variations in the data obtained by the coincidence method. The dashed horizontal line represents the authors' judgment regarding the proper interpretation of their data. The consistency of the position of the points with respect to the dashed line is *not such*, however, *as to exclude* the possibility of 10 or 20 percent deviations from constancy of σ_θ for values of θ between 15° and 90°.

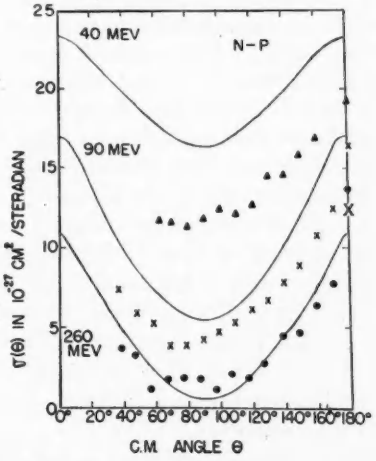

FIG. 7. Same as Fig. 12 of CSW (reference 44). Points are plotted from measurements of Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949) and of Kelly, Leith, Segrè, and Wiegand, Phys. Rev. **79**, 96 (1950). The large X has been communicated privately by R. H. Fox to CSW. Estimated uncertainty of experimental values is 20 percent. Potentials used in calculations of the theoretical curves by Christian and Hart (reference 47) are

$$V_{\text{singlet}}=(-35.3\text{ Mev})(\tfrac{1}{2}+\tfrac{1}{2}P_M)\mathcal{Y}(r/r_0),$$

$$V_{\text{triplet}}=\{(-25.3\text{ Mev})(\tfrac{1}{2}+\tfrac{1}{2}P_M)+(-48.2\text{ Mev})(0.37+0.63P_M)S_{12}\}\mathcal{Y}(r/r_0);$$

P_M = Majorana exchange operator;

$$S_{12}=\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2);\quad r_0=1.35\times10^{-13}\text{ cm}.$$

[46] Emilio Segrè and Owen Chamberlain (private communication).

In Fig. 6 is reproduced the Berkeley information regarding σ_θ at $\theta=90°$. In Fig. 7 are shown $n-p$ results at $E=40$, 90, 260 Mev together with theoretical curves of Christian and Hart.[47]

In a more recent paper Segrè[45] gives a revised version of the data presented here in Fig. 5. These data are plotted in Fig. 5 of his paper and a collection of total cross section measurements is found in Table II, page 89 of his paper. It will be noted that the three graphs of Segrè's Fig. 5 are drawn through the experimental points and that they do not show a marked fore and aft asymmetry. At 90 Mev it would be difficult, however, to pass a curve through the experimental points without having ~10 or 20 percent more scattering per unit solid angle at 180° than at 0°. Guernsey, Mott, and Nelson[48] have measured $n-p$ scattering at Rochester for a neutron energy near 220 Mev. Their detector system consisted of a counter telescope of four anthracene crystal counters, and the energy of the recoil protons was measured by means of a pulse-height analysis for the last counter. In this way the neutron energy could be known to a few percent by a suitable selection of pulse heights. Since their incident neutron beam was spread in energy from 150–250 Mev, it was possible to obtain the $n-p$ cross section at other energies in this range, and they published data at 180 Mev as an illustration of the method. Their 220-Mev results, in the angular range $75°<\theta\leqslant 180°$ for the scattered neutron in the center-of-mass system, agree with those obtained at 260 Mev at Berkeley in both shape and magnitude. The Christian-Hart theoretical curve for a Yukawa potential with tensor force calculated for 280-Mev incident neutron energy agrees equally well with both experiments.

There is a striking difference in the character of the $p-p$ and $n-p$ results. The latter show a marked minimum at $\theta=90°$ and σ_θ is not even approximately constant.

The work of Birge, Kruse, and Ramsey[49] (hereafter referred to as BKR) with the Harvard cyclotron was done at $E=75$ and 105 Mev. The *internal* cyclotron *beam* was used. Anthracene rather than stilbene crystals were used for counting.

It is stated by the Harvard workers that the proton energy was measured to be 75 Mev with a 7-Mev half-width at 35 inches from the cyclotron center and 105 Mev with an 8.5-Mev half-width at 40 inches. A probe between the cyclotron pole pieces in the positions just mentioned determined the energy of the protons. The shape of the energy distribution curve is not given and the degree of masking of the angular distribution arising from the wings of the energy distribution curve is not obvious. The measured cross section is practically a constant for $40°<\theta<90°$ and has the values 6.6 mb/sterad and 5.4 mb/sterad at 75 Mev and 105 Mev, respectively. The absolute value depends on the knowledge of the cross section of $C^{12}(p, pn)C^{11}$ reaction which must depend in turn on the measurement of the primary beam intensity.

The experimental results show deviations from an angle-independent σ_θ at $\theta=40°$, 50°, and 80° for $E=75$ Mev and for $\theta=50°$ and 80° at $E=105$ Mev. At 75 Mev the deviations at 40° and 50° are appreciably greater than the statistical errors recorded by BKR in their Fig. 5. The other deviations are barely on the limit of the statistical error. The latter is of the order of 0.1 mb/sterad. One of the deviations is of the order of 0.4 mb/sterad. The values of σ_θ at 105 Mev are about $\frac{1}{6}$ mb/sterad higher than expected from the Berkeley measurements. The Harvard workers point to their procedure in measuring the primary beam current as a possible reason for the discrepancy and hope to obtain a more definite answer regarding absolute values in experiments with an external beam. As in the Berkeley work a literal interpretation of angular distributions within the limits of statistical error would be risky; similarly the conclusion that indications of lack of constancy of σ_θ under variations of θ lying outside the statistical error of σ_θ are unreal are perhaps also unjustifiable. The horizontal dotted lines used by both groups as a schematic representation of the dependence of σ_θ on θ are perhaps an oversimplification.

A thorough description of an experiment at $E=240$ Mev has been recently given by Oxley

[47] R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950).
[48] Guernsey, Mott, and Nelson, Phys. Rev. (to be published).
[49] Birge, Kruse, and Ramsey, Phys. Rev. **83**, 274 (1951).

and Schamberger.[50] The work was done in the internal beam of the Rochester synchrocyclotron. The characteristic features of this work are as follows. *Primary beam current* measurement was made by observing the beta-activity of C^{11} produced in $C^{12}(p, pn)C^{11}$ as in the Harvard work. The C^{11} cross section was obtained from the measurements of Aamodt, Peterson, and Phillips.[51] *An independent partial check* was obtained by measuring the $p-p$ cross section employing the C^{11} method both for the primary beam and the scattered proton measurements. The fact that the authors check their own results appears significant in view of their disagreement with Berkeley regarding the absolute value. *The primary beam energy* was computed to be 243 Mev from orbit dimensions and was measured as (235 ± 10) Mev by absorption in Cu. *The primary beam energy spread* was measured photographically to be 20 Mev at half-maximum.

The result of this work is $\sigma_\theta=(4.97\pm0.43)$ mb/sterad in the range $26.8°\leqslant\theta\leqslant90°$. Combined statistical and geometrical errors are considered to give an uncertainty of 3.5 to 5.5 percent in relative values for different θ and an uncertainty of 8.4 percent is believed to exist besides on account of the employment of C^{11} for primary beam current measurements. The result is 29 percent higher than that of the Berkeley group at the same energy. The discrepancy appears to be higher than the combined uncertainties of 8.8 and 8.6 percent assigned to their results by the Berkeley and Rochester groups. The cross section is independent of angle within the relative error ($\sim$4.5 percent).

The conclusions of Oxley and Schamberger[50] are checked in all essential respects by the work of Towler,[52] who employed a magnetic deflection method following a suggestion of Oxley's. This work of Towler[52] is likely to prove significant for the following reasons. (1) It provides a check on counter measurements by an independent procedure. (2) The observations are made on the low energy partner of the scattered-recoil pair providing an interesting variation in experimental procedure. (3) Observations on polyethylene have been compared with those for polystyrene targets, providing a valuable check on the procedure used in subtracting the carbon background. (4) In absolute value the results agree more closely with the observations of Oxley and Schamberger[50] and of Birge, Kruse, and Ramsey[49] than of the Berkeley workers. (5) They corroborate the existence of a large rise in σ at small angles which is evident in the curves of Chamberlain, Segrè, and Wiegand.[44] Towler's results give a practically constant σ_θ for θ between 13.0° and 71.9° with a mean value of (4.66 ± 0.39) mb/sterad; at $\theta=8.7°$ the value of σ suddenly rises to (15.6 ± 1.6) mb/sterad. This rise is of a magnitude which admits an explanation in terms of Coulomb scattering and its interference with specifically nuclear scattering. It may be hoped that this circumstance will be of help in determining the presence of waves with $L>0$ with the aid of the Coulomb wave as a reference standard.

Proton-proton scattering experiments at $E=146$ Mev have been reported by Cassels.[53] These were performed in the external beam of the 110-inch Harwell cyclotron. Except for the smallest scattering angle (25°) the Wilson-Creutz coincidence technique[54] was used for counting scattered protons. Cassels' results show a practically angle independent $\sigma_\theta=(4.86\pm0.25)$ mb/sterad at five scattering angles between $\theta=35°$ and 90°. At $\theta=25°$ his value of σ_θ is about 5.5 mb/sterad. He considers this point on his curve as less reliable than the others, since only one counter was used and since the background correction was consequently difficult to make. It is noteworthy that the Berkeley value of σ_θ for larger scattering angles of $\sim$3.9 mb/sterad is lower than Cassels' value by roughly the same fractional amount as it differs from the Rochester and Harvard values.

A consideration of the above experiments indicates agreement among observers regarding the main features: approximate independence on angle for the larger scattering angles and approximate independence on energy. There is also some agreement regarding the existence of a rise in σ_θ at small θ, notably between the Berkeley and Rochester observers. Cassels' ob-

[50] C. L. Oxley and R. D. Schamberger, Phys. Rev. **85**, 416 (1952).
[51] Aamodt, Peterson, and Phillips, UCRL 526 (1949).
[52] O. A. Towler, Jr., Phys. Rev. **85**, 1024 (1952).
[53] J. M. Cassels, Proc. Int. Conf. on Nuc. Phys. and Phys. of Fund. Part., p. 95, September (1951).
[54] R. R. Wilson and E. G. Creutz, Phys. Rev. **71**, 339 (1947).

servation at 25° points in the same direction but in his opinion this observation should be used with caution.

Interpretation of Scattering Results

The features of high energy scattering occupying the center of interest are sufficiently different from those of the low energy region to justify a separate discussion. The presentation will be facilitated by a brief review of some of the older work.

A classification of types of interactions has been introduced by Rarita and Schwinger[55] and is frequently used in the literature. The original terminology has been suggested by forms of meson theories giving rise to the interactions. These types are the symmetrical interaction, suggested by a meson theory in which charged particles of both signs as well as neutral ones are permitted, the charged interaction and the neutral interaction, where only charged or only neutral mesons are used. The designations "Exchange" interaction and "Ordinary" interaction have been subsequently introduced in place of "Charged" and "Neutral," respectively, by Camac and Bethe[56] and by Wu and Ashkin,[57] the latter in connection with $n-d$ scattering. Ashkin and Wu[58] consider the effect of the tensor force and calculate neutron-proton as well as proton-proton scattering for incident energies from 100 to 200 Mev. The general conclusion reached by comparison with experimental results of McMillan and co-workers[59] is that exchange and symmetric theories give too high a value of $\sigma(180°)/\sigma(90°)$ while ordinary forces give an angular distribution which is too strongly forward. The experimental distribution is approximately symmetric about 90°, suggesting the form

$$V=\tfrac{1}{2}(1+P_M)J(\mathbf{r}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$$

for the interaction potential. Here P_M is the Majorana exchange operator. This interaction was first proposed by Serber and is referred to as the even or Serber interaction. For it, odd orbital angular momenta do not give rise to scattering except for interference terms with even L.

Other calculations along these lines have been made by Barker[60] and Chew and Goldberger[61] with results similar to those of Camac and Bethe; by Eisenstein and Rohrlich[62] and Rohrlich and Eisenstein,[63] which favor the symmetrical interaction although with a reduced tensor force range contrary to indications from low energy data; by Massey, Burhop, and Hu,[64] who conclude that simultaneous fitting of $n-p$ angular distribution and total cross section is difficult or impossible; and by Hu and Massey[65] and Burhop and Yadav,[66] who do not succeed in accounting for experimental results.

Data for $E>100$ Mev together with that at lower energies have been discussed theoretically by Christian and Hart,[47] Christian and Noyes,[67] Case and Pais,[68] and by Jastrow.[69] Christian and Hart analyze $n-p$ data from $E=40$ to 90 Mev on the basis of the $\frac{1}{2}(1+P_M)$ exchange potential. Their work falls outside the somewhat arbitrary Rarita-Schwinger classification of potentials according to relationship to form of meson theory. Their results are as follows. (1) Assuming the potential to have the same radial dependence in all states, the value of a in $\frac{1}{2}(1-a+aP_M)$ is probably between 0.50 and 0.60. (2) A long-tailed potential is called for by the presence of phase shifts for high L at 40 Mev. The square and Gauss potentials are less probable than the exponential and Yukawa potentials. The latter two are acceptable within experimental uncertainties. (3) The angular distribution fit is improved by the inclusion of a tensor interaction which may be allowed to have a larger range than that of the central force.

[55] W. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941).
[56] M. Camac and H. A. Bethe, Phys. Rev. **73**, 191 (1948).
[57] T. Y. Wu and J. Ashkin, Phys. Rev. **73**, 986 (1948).
[58] J. Ashkin and T. Y. Wu, Phys. Rev. **73**, 973 (1948).
[59] Cook, McMillan, Peterson, and Sewell, Phys. Rev. **72**, 1264 (1947).
[60] F. C. Barker, Nature **151**, 726 (1948).
[61] G. F. Chew and M. L. Goldberger, Phys. Rev. **73**, 1408 (1948).
[62] J. Eisenstein and F. Rohrlich, Phys. Rev. **73**, 641 (1948).
[63] F. Rohrlich and J. Eisenstein, Phys. Rev. **75**, 705 (1949).
[64] Massey, Burhop, and Hu, Phys. Rev. **73**, 1403 (1948).
[65] T. M. Hu and H. S. W. Massey, Proc. Roy. Soc. (London) **A196**, 135 (1949).
[66] E. H. S. Burhop and H. H. Yadav, Proc. Roy. Soc. (London) **A197**, 505 (1949).
[67] R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1950).
[68] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).
[69] R. Jastrow, Phys. Rev. **79**, 389 (1950) and **81**, 165 (1951).

Christian and Noyes[67] attempt to account for $p-p$ scattering at 32 and at 340 Mev and $n-p$ scattering at 40, 90, 280 Mev by means of static potentials. They claim that it is not possible to do so without giving up the assumption of charge independence in interactions between nucleons. This conclusion has been disagreed with by Case and Pais.[68] The other conclusions of Christian and Noyes are as follows. (1) Data on $p-p$ scattering at 32 Mev can be explained by two combinations of tensor and central force for both of which the tensor potential has a singularity at $r=0$ either of the type $1/r$ or $1/r^2$. The central potential is not long-tailed. The best fit is with a singlet square well of range 2.6×10^{-13} cm and a tensor potential $V_t\mathfrak{y}(r/R)$, $V_t=\pm23$ Mev, $R=1.25\times10^{-13}$ cm or else the same square well and a tensor potential $V_t\mathfrak{y}(r/R)/(r/R)$ with $V_t=\pm18$ Mev, $R=1.6\times10^{-13}$ cm. (2) The fit to 340 Mev is attained only by means of the latter type of singular tensor interaction. (3) The best $p-p$ fit attributes about $\frac{1}{3}$ of the scattering at 32 Mev to the tensor force. (4) A general argument, based on analogy to hard sphere scattering, is developed according to which the region from $r=0$ to 0.5×10^{-13} cm must be brought into play in order to explain $p-p$ scattering at 340 Mev. This argument serves as the justification for the employment of singular tensor potentials. The effect of the singular tensor potential of Christian and Noyes on low energy $p-p$ scattering has been investigated by Crosbie,[70] who finds an appreciable theoretical effect below 10 Mev. Comparison with data of Rouvina[15] at 6.85 Mev shows that the theoretical cross section, with singular tensor force scattering included, does not agree as well as pure S wave scattering, but the disagreement is not outside the experimental uncertainties.

It should be noted here that the word "singular" as used by Christian and Noyes and by Christian and Hart differs from the usual mathematical meaning. The "singular" potentials would be singular if they were used at $r=0$. What is really meant is that the potential is made to vary rapidly in the *intermediate* range of values of r.

Case and Pais[68] arrive at a conclusion regarding charge independence of nuclear forces which is the opposite of that reached by Christian and Noyes.[67] There is no proof of the impossibility of preserving this simplifying assumption in Christian and Noyes' paper, the argument being mainly one of plausibility on inspection of $p-p$ and $n-p$ data. There is agreement between the two views regarding the fact that $p-p$ data require emphasis on short distance interactions, but in effect Case and Pais[68] point out that the states present in $p-p$ scattering could conceivably be more sensitive to whatever happens at short distances than the states $({}^3S, {}^1P, \cdots)$ which are present only in $n-p$ scattering. Since among these ${}^3S$ has a large statistical weight and is the most important at 30 Mev, the presence of $({}^1S, {}^3P, \cdots)$ states in $n-p$ scattering is only of secondary importance. The additional interaction considered by Case and Pais is represented by an addition to the potential energy of amount

$$V(r)(\mathbf{L}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)),$$

where L is the relative orbital angular momentum vector. For $p-p$ scattering they use no additional interaction in triplet states and explain the rise in cross section toward 90° as the effect of three unequal phase shifts for ${}^3P_{0,1,2}$. These are in the ratios

$${}^3K_1{}^{(2)}:{}^3K_1{}^{(1)}:{}^3K_1{}^{(0)}=1:(-1):(-2),$$

where the multiplicity is indicated on the left, L as a subscript, and the total angular momentum on the upper right. For 30 Mev they find ${}^3K_1{}^{(2)}\cong3.7°$. The sign of this phase shift corresponds to attraction between the protons and fits the direction desired in the considerations of shell formation in heavy nuclei. By means of the first Born approximation, Case and Pais[68] show that terms in $\mathbf{L}\cdot\mathbf{S}$ have the property of raising 90° scattering and that the main features of $p-p$ scattering at 350 Mev are not in contradiction with an interaction composed of an even part of a term in $\mathbf{L}\cdot\mathbf{S}$. A best fit is not presented by them because qualitative considerations show that the results are sensitive to the shape and range of the factor φ_2 which multiplies $\mathbf{L}\cdot\mathbf{S}$. Two general features of the 350 Mev results are considered. In the first place, the ratio of 90° scattering cross sections at 350 and 30 Mev has to be accounted for. The singlet

[70] E. A. Crosbie, U. S. Office of Naval Research Report from University of Pittsburgh.

scattering at 350 Mev is small, ~0.2 mb, and they have to explain practically the whole experimental value of ~4 mb on the basis of the term in $\mathbf{L}\cdot\mathbf{S}$. This ratio is very sensitive to assumed changes in radial dependence of φ_2. Secondly, the flatness of the σ_θ, θ curve has to be explained. Here they find an even more striking sensitivity to radial dependence of φ_2, obtaining the values of $\sigma(30°)/\sigma(90°)$ given in Table IV.

TABLE IV. Values of $\sigma(30°)/\sigma(90°)$ at 350-Mev $p-p$ scattering for different shapes of factor multiplying $\mathbf{L}\cdot\mathbf{S}$.

Exponential	Yukawa $r^{-2}\exp(-r/r_0)$	$C\frac{d\mathcal{Y}(x)}{xdx}$	Square well
930	40	4.7	1.6	0.42

They expect that for $d\mathcal{Y}/xdx$ there will be approximate energy independence of $\sigma(90°)$. Later observations are at least not in disagreement with this expectation.

They also have been able to produce reasonable arguments to the effect that new bound states of the deuteron will not be produced. Their arguments for considering $n-p$ scattering as being not in disagreement with the theoretical view proposed is mainly that the contribution to 90° scattering caused by the $\mathbf{L}\cdot\mathbf{S}$ terms is estimated to be about 4 times as large in the $p-p$ case as in the $n-p$. From $p-p$ data they estimate therefore that $\sigma_{LS}{}^{n-p}(90°)$ is ~1 mb at $E=260$ Mev, and this amount appears to be reconcilable with experiment.

Case of Pais[58] also considered the interference of p and s waves. Since the p phase shifts are small the first-order effects cancel, and they obtain a contribution having spherical symmetry in the center-of-mass system. A note in proof states that Christian has pointed out that the interference with d waves does not give rise to a spherically symmetric contribution to $n-p$ scattering. Case and Pais find that this interference gives rise to an asymmetry in the $n-p$ scattering which has a direction opposite to that observed and that the sign of φ_2 has to be changed so that the connection with "shell theory" is lost. Similarly there is difficulty in making a simultaneous fit to the 30- and 350-Mev data.

It may be pointed out that even apart from the possibility of tensor forces contributing appreciably[71] to the $j-j$ coupling conditions in heavy nuclei, the energy with which the nucleons collide is so different from that encountered in an average heavy nucleus that there need not be a direct connection of sign in the two cases, different internucleon distances assuming different relative importance. An explanation along the lines of Case and Pais is only a schematized way of taking into account couplings of the type

$$\mathbf{A}_1\cdot\boldsymbol{\sigma}_1+\mathbf{A}_2\cdot\boldsymbol{\sigma}_2,$$

where $\mathbf{A}_1$, $\mathbf{A}_2$ are orbital matrix vectors. With such an interaction the possibilities of sign reversal are more natural than with the specialized form considered by Case and Pais.

Another way of preserving charge independence has been found by Jastrow,[69] who points out that one can explain the main features of high energy scattering in terms of a potential with a repulsive core. The possibility of a repulsive core has been advocated from the viewpoint of saturation of nuclear forces by Kramers and Bethe.[72] High energy $p-p$ scattering in the work of Jastrow appears to give an indication of its existence but curiously only for interactions in singlet states and not with the same radius of repulsive core in triplet states. In the final fit the triplet core radius is in fact taken to be zero.

Jastrow's considerations are carried out for a potential of the form:

Singlet

$$V=\infty, \quad (r<r_0)$$

$$V=V_{0s}\frac{1+P_M}{2}\exp\left(-\frac{r-r_0}{r_s}\right), \quad (r>r_0)$$

Triplet

$$V=\{a+(1-a)P_M+[b+(1-b)P_M]\bar{\gamma}S_{12}\}V_{0t}\exp(-r/r_t).$$

One of his better fits corresponds to

$r_0=0.60\times10^{-13}$ cm	$V_{0s}=375$ Mev
$r_s=0.40\times10^{-13}$ cm	$V_{0t}=69$ Mev
$r_t=0.75\times10^{-13}$ cm	$\bar{\gamma}=1.84$
$a=0.50$	$b=0.30$.

[71] A. M. Feingold and E. Wigner, Phys. Rev. **79**, 221 (1950).
[72] H. A. Kramers and H. A. Bethe, Ann Arbor Summer Physics Symposium, 1935.

The action of the repulsive core at 340 Mev for $p-p$ scattering is principally that the ^1S phase shift becomes negative on account of the strong penetration of the s wave into the core. On the other hand, the ^1D wave is positive, since the d wave does not penetrate to small distances as readily and is mainly exposed to the attractive part of the potential outside the core. There results a rise of σ at $\theta=90°$ and a dip at $\sim 40°$. The tensor triplet interaction contributes enough in the region of the dip so that a flat σ, θ curve results.

In the region of $E \cong 30$ Mev the main problem is that of accounting for a rise in σ_θ from ~ 12 mb at $\theta=20°$ to ~ 15 mb at $\theta=90°$. A large ^1D wave is a disadvantage in explaining such a rise because a positive ^1D phase shift gives, with a positive ^1S phase shift, a negative interference term for σ. At 30 Mev the repulsive core diminishes the expected ^1D phase shift because the exponential used has a smaller range constant and a larger depth constant than it would have without the repulsive core. This circumstance results from fitting low energy data as may be seen in the following manner.

In making low energy fits one has to reproduce the same value of $r\partial\mathfrak{F}/\mathfrak{F}\partial r$ and of $\partial(\partial\mathfrak{F}/\mathfrak{F}\partial r)/\partial E$ at some value of E close to $E=0$. The first requirement is equivalent to the reproduction of the scattering length, the second is often spoken of as the adjustment of the effective range, but is actually very simple. Since $\partial(\partial\mathfrak{F}/\mathfrak{F}\partial r)/\partial E$ is of the form of a positive constant multiplied by $[-\mathfrak{F}^{-2}\int_0^r \mathfrak{F}^2 dr]$, the comparison of the $\partial/\partial E$ quantity for the same value of r with core and without shows that the effect of the core is opposite to that of concentrating the attractive region at small distances. The effect of the core is thus similar to an increase in the range parameter. The matter appears even more simply in terms of percentage changes of the effective kinetic energy. On account of the core, the adjustment to scattering length requires the employment of more attraction at larger distances. The percentage change of the effective kinetic energy by changing E becomes therefore small in the attractive region, and E has little effect on $\partial\mathfrak{F}/\mathfrak{F}\partial r$. On the other hand, attraction concentrated at small distances when compared with the "core plus attractive potential" at the same r leaves a region of r in which the percentage change of effective kinetic energy is large for the same change in E so that $\partial\mathfrak{F}/\mathfrak{F}\partial r$ is sensitive to energy.

On account of the necessity of fitting low energy data, the core requires a reduction in the range constant and consequently a diminution in the tail with a consequent decrease of phase shifts with larger L. The undesirable effect of the $s-d$ interference is accordingly decreased, the d phase shift having dropped in Jastrow's calculations from 1.2° to 0.6°. The s wave phase shift at 30 Mev is also decreased by the introduction of the core and the average decrease in σ is compensated by the tensor interaction. These changes give an excellent fit to the angular distribution at $E=30$ Mev, the Coulomb scattering playing an important part. Pure Coulomb scattering at $\theta=20°$, $\Theta=10°$, $E=30$ Mev gives $\sigma \sim 6.1$ mb/sterad. The observed value is ~ 12 mb/sterad.

A reader of Jastrow's paper can easily form the impression that the Coulomb wave has been neglected at all energies. Such a neglect would not be justifiable at 30 Mev and has not been made.

Increasing the core radius has the effect of decreasing the 90° cross section at 340 Mev and consequently a decrease in tensor strength is called for bringing with it a less complete filling of the dip in the σ, θ plot close to 50°. The two opposing requirements interfere with the reproduction of the experimental scattering isotropy. It would not be surprising, however, if the velocity dependence of nuclear forces were large enough to account for changes of the order of the discrepancy.

The ^1S phase shift goes through 0 at $E \cong 150$ Mev and $\sigma(90°)$ is consequently expected to have a minimum at about this energy. The Harvard, Rochester, Harwell data are at least not in disagreement with this expectation.

Agreement with $n-p$ data as secured in Jastrow's work appears via the Christian-Hart[47] fit. The triplet interaction is assumed to have no repulsive core. The statistical weight $\frac{1}{4}$ makes the states which are most important in Jastrow's $p-p$ fit have secondary importance for $n-p$ scattering. The fits are on the whole better at 90 and 240 Mev than at 40 Mev. The ratio

$\sigma(\pi)/\sigma(\pi/2)$ is 1.55 experimentally and 1.3 theoretically for 40 Mev.

The possibility of a repulsive interaction in triplet states has been considered by Parzen and Schiff[73] and by Olsson.[74] Parzen and Schiff have found that an excessive density of heavy nuclei may be expected as a result of a repulsive core in triplet states. Olsson calculated the consequences of the hard core model for $n-p$ scattering between 38 and 140 Mev. He obtained agreement with experiment at low energies and was able to reproduce the experimental σ approximately at 90 Mev employing the symmetric interaction. At the high energies, however, the calculated values were about three times the observed.

The following general comments regarding the repulsive core hypothesis, as used in Jastrow's first set of papers, appear appropriate. (1) The introduction of the core has the effect of decreasing the range parameter of the singlet interaction. Jastrow's calculations are carried out for the exponential potential and consequences for the apparent meson mass are not directly obvious. An estimate shows,[75] however, that the meson mass needed to reproduce the attractive region of the corresponding Yukawa potential corresponds to $\mu=450\ m$. The validity of the meson theory of nuclear forces comes less into question for the representation of the tail of the potential energy curve and the disagreement with the observed mass of the π-meson is therefore more significant with the repulsive core than without it. (2) As has been brought out by Jastrow,[69] the core radius used by him is so small that the effect of the repulsion on saturation of nuclear forces may not be sufficiently pronounced to secure saturation without the aid of other devices such as exchange forces. (3) The range of the tensor force is not likely to be the same as that of the central interaction. Appreciable differences are indicated by the work of Guindon,[76] Broyles and Hull,[77] Pease and Feshbach,[78] Feshbach and Schwinger.[9] (4) There is so far no evidence from meson phenomena which indicates the existence of a meson of much higher mass which can be held responsible for a very short range repulsion.

The above comments regarding Jastrow's work may appear to be largely critical. On the positive side of the argument one has the following considerations. (1) The hypothesis of charge independence of nuclear forces can be kept as a simplifying first approximation. (2) There is an appealing connection with saturation of nuclear forces. However, this side of the matter is more involved than would appear at first sight. (3) Fits to experiment can perhaps be improved by combining the repulsive core with interactions of the $\mathbf{A}_1\cdot\boldsymbol{\sigma}_1+\mathbf{A}_2\cdot\boldsymbol{\sigma}_2$ type. The presence of such interactions is indicated by $j-j$ coupling in heavy nuclei. There is the possibility, however, that strong coupling of $\mathbf{L}$ to $\mathbf{S}$ is a many-nucleon phenomenon.

The angular isotropy at high energies suggests either a superposition of phase shifts or else a radical departure from the low energy condition. A possibility of the latter type has been recently suggested,[75,79] the question having been raised as to whether protons have isomers. The formation of an isomer of one of the protons in the final state can be helpful in accounting for the facts provided it can be produced from states other than ^1S of the incident wave. Conservation of angular momentum and of parity are both satisfied if the isomer is odd and has spin $\frac{1}{2}$ since then the $^3P_1$ state of the incident wave can produce a ^3S state. In order to maintain this explanation, it would be necessary to suppose that transitions to an isomeric state are very improbable except in high energy collisions. Towler's[52] results and their agreement with the findings of Segrè *et al.*, regarding the maximum at low energies make the isomeric state explanation less attractive. Together with the Berkeley measurements they suggest that the Coulomb field interference with specifically nuclear scat-

[73] G. Parzen and L. I. Schiff, Phys. Rev. **74**, 1564 (1948).
[74] P. O. Olsson, Phys. Rev. **83**, 845 (1952).
[75] G. Breit, Proceedings of the International Conference on Nuclear Physics and Fundamental Particles, p. 102, September, 1951 (more accurate and detailed calculations of M. H. Hull and A. Herschman confirm this result). It is of interest to note here, however, that subsequent work by Lévy and Lepore and Jastrow would interpret the large effective value of μ as a consequence of fourth-order interactions which make the potential decrease more rapidly with distance outside the core.
[76] W. G. Guindon, Phys. Rev. **74**, 145 (1948).
[77] A. A. Broyles and M. H. Hull, Phys. Rev. **79**, 247 (1950).
[78] R. L. Pease and H. Feshbach, Phys. Rev. **81**, 142 (1951).
[79] G. Breit, Phys. Rev. **84**, 1053 (1951); Bull. Am. Phys. Soc. **27**, 32 (1952); G. Breit and H. M. Jones, Phys. Rev. **84**, 1054 (1951).

tering is at least partly responsible for the apparent isotropy.

The discussion of Jastrow's work in the present section will be supplemented in a later one by the consideration of the "contact terms" of the PS (ps) theory as developed by Lévy.

MESON THEORY OF NUCLEAR FORCES

In the present section a brief account will be given of developments in the meson theory of nuclear forces which have been made along the more standard lines of field theoretic considerations. The difficult question of whether or not the types of meson theory commonly used are even approximately on the right track is purposely not emphasized.

The original Yukawa[25] idea was modeled along the lines of electromagnetic theory. The introduction of a particle having a finite rather than zero mass resulted in furnishing an exponential factor in the formula for the potential energy. This factor

$$e^{-x}, \quad x=r/a, \quad a=\hbar/\mu c$$

contains the meson mass μ which determines the range parameter a. For the ^1S interaction many modifications of the theory give a potential energy of the form

$$\mathcal{C}e^{-x}/x.$$

This form of the potential energy is not directly in contradiction with low energy experiments as long as the value of a is not required to be in exactly the right relationship to the meson mass μ. One can even claim that it agrees with instinctive desires for simplicity of interpretation somewhat better than other phenomenologic potentials such as the square well or the Gauss error potentials. Thus the "long tail" property of this form of interaction energy fits in nicely with a desire to make the approximately established equality of $n-p$ and $p-p$ forces the same in ^1S states, and it has been used successfully in the recent work of Bethe,[36] Bethe and Longmire,[10] Salpeter,[3] Feshbach and Schwinger,[9] Pease and Feshbach[18] in the treatment of the binding energy of the deuteron and triton, thermal neutron scattering lengths, intermediate energy $n-p$ and $p-p$ scattering and the photodisintegration of the deuteron. It is generally realized that no decision regarding the correctness of the potential is likely to result from such studies alone. The main stumbling block lies in the superabundance of parameters present in the potentials representing forces in ^3S, ^3P, and other states. There is, consequently, some difficulty in determining from experiment anything more than a condition which must be satisfied by the parameters. The form of the potential energy for these states is more complicated than the expression for the ^1S potential and the depth, range, and shape specification of the tensor force containing the spin dependent factor

$$S_{12}=3(\sigma_1\mathbf{r})(\sigma_2\mathbf{r})/r^2-(\sigma_1\sigma_2)$$

adds further parameters.

There is no doubt concerning the usefulness of the experimental information on all of the above phenomena, and the theoretical attempts at interpretation have contributed to the ruling out of many possibilities. The experience so far has been, however, that these considerations are not able to determine the interactions or the best type of meson theory. In addition to the superabundance of parameters there is the troublesome fact that the meson theories are subject to divergence difficulties. In many cases this circumstance necessitated arbitrary remedies such as short range cutoffs. The development of renormalization procedures which have proved successful in electrodynamics has met with difficulty in meson theory.

Some of the assumed forms have indeed proved to be nonrenormalizable by the means tried. Fortunately the pseudoscalar theory with pseudoscalar coupling is renormalizable and fits the experimentally established pseudoscalar character of the pi-meson. On the other hand the coupling constant $f^2/4\pi$ (Lorentz-Heaviside units) is of the order of 10, and the weak coupling form of the theory, which is the only form for which detailed calculations have been made, shows no evidence of convergence as a power series in f. The practical problem of fitting meson theories to facts about nuclear forces is thus complicated by the absence of a logically self-consistent form of meson theory; the arbitrary modifications of the divergent results of meson theories have furnished an almost infinite variety of modifications of meson theoretic formulas

which can be considered as competitors. If one wished to be cynical, one could emphasize a similarity between styles in women's hats and styles in meson theories. On the other hand, the pseudoscalar character of the pi-meson has been established only recently. The rapid advances in the experimental exploration of meson phenomena by means of high energy accelerators and the theoretical interpretation of such results are promising to eliminate many of the outstanding uncertainties. The following advances appear to be especially noteworthy:

The discovery[80] of the π-meson and associated phenomena makes it overwhelmingly likely that nuclear forces are connected with π-mesons rather than μ-mesons. It is now known that π-mesons are of three types π^+, π^-, π^0, carrying charges $+e$, $-e$, 0, respectively. Here e is the charge on the proton. The first two types are called charged, the last—neutral. These particles are often referred to as *pions*. Application of detailed balancing arguments[81] to experimental results[82] on

$$\pi^+ + d \rightleftarrows p + p$$

shows that the spin of π^+ is 0. The intrinsic parity of the π^- meson has been established to be negative on the basis of experiments on the capture of π^- meson by deuterium giving the reaction

$$\pi^- + d \rightarrow n + n.$$

This reaction takes place predominantly through the intermediate state of a π^- meson having been captured in a K orbit. Since the spin of the π^- mesons is supposed to be zero just like the spin of π^+ the total initial angular momentum of the system is that of the deuteron and is 1 in units $\hbar$. The final angular momentum is therefore also 1. On account of Pauli's exclusion principle, this value of the angular momentum can be obtained only in the $^3P_1$ state and the fact that the reaction takes place shows therefore that the parity of the initial state is odd.[83] Since the parity of the deuteron is even the parity of π^- must be odd. Thus from the assumption that π^+ and π^- have the same spin and from observations on the absorption of π^- in deuterium it follows that π^- is pseudoscalar.

The scattering of π^+ and π^- mesons in hydrogen has been studied with special thoroughness by Fermi, Anderson, and Nagle.[84] In this work there has been no experimental indication which would make one suspect the assumption that the spins of π^+ and π^- are the same. The well-known facts pointing toward the symmetry of the nuclear Hamiltonian form some evidence for the identity of spin and parity of π^+ and π^-. The spin of π^0 has been shown[85] to be zero making use of the observation that

$$\pi^0 \rightarrow \gamma + \gamma.$$

In view of these and other facts and other evidence summarized by Marshak,[81] it is generally believed that the three kinds of π-meson have spin 0 and odd parity. Such particles are generally referred to as pseudoscalar and the associated field is spoken of as a pseudoscalar field. Previously to these advances in meson physics the possibilities were more numerous, other types of fields such as the vector, scalar, pseudovector fields having been under consideration. Danysz, Lock, and Yekutieli[86] have recently presented evidence, based on analyses of angular and energy correlations of pairs of cosmic-ray shower particles identified as π-mesons, suggesting that in some high energy nuclear disintegrations the π-mesons are a secondary product of the reaction resulting from the decay of a neutral particle of mass 556 m_e and lifetime of 10^{-14} sec. Kaplon and Ritson[87] looked for evidence for the existence of this particle in shower particles having much higher

[80] Lattes, Occhialini, and Powell, Nature **160**, 453, 486 (1947).

[81] R. E. Marshak, Rochester High Energy Conference, December (1950); W. B. Cheston, Phys. Rev. **83**, 1118 (1951); R. E. Marshak, Revs. Modern Phys. **23**, 137 (1951); M. H. Johnson, private communication to Clark, Roberts, and Wilson.

[82] Durbin, Loar, and Steinberger, Phys. Rev. **83**, 646 (1951); Clark, Roberts, and Wilson, Phys. Rev. **83**, 649 (1951).

[83] S. Tamor and R. E. Marshak, Phys. Rev. **80**, 766 (1950); Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951); S. Tamor, Phys. Rev. **82**, 38 (1951); Ogawa, Yamada, and Magahara, Prog. Theoret. Phys. **6**, 227 (1951).

[84] Fermi, Anderson, and Nagle, *et al.*, Phys. Rev. **85**, 934, 935, 936 (1952); K. A. Brueckner, Phys. Rev. **86**, 626 (1952); Isaacs, Sachs, and Steinberger, Phys. Rev. **85**, 803 (1952).

[85] C. N. Yang, Phys. Rev. **77**, 242 (1950); Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950); J. Schwinger, Phys. Rev. **82**, 664 (1951).

[86] Danysz, Lock, and Yekutieli, Nature **169**, 364 (1952).

[87] M. F. Kaplon and D. M. Ritson, Phys. Rev. **87**, 664 (1952).

energies than those discussed by Danysz *et al.* They concluded that most of the correlated pairs they observed could be due to the positron-electron pairs produced in π^0 decay. Thus 31 pairs observed by Kaplon and Ritson could have been from decay of the ζ^0 meson suggested by Danysz, *et al.* but 26 positron-electron pairs might have been expected in the conditions of the experiment. Kaplon and Ritson did not feel compelled to introduce the ζ^0 meson to explain their data. Brueckner and Watson[88] have suggested that the data of Danysz *et al.*, could be considered as evidence for a meson-meson interaction. A similar suggestion has been made, without publication, by Fermi. This interaction is supposed to produce the observed correlations after production of the π-mesons in the high energy primary event. Such effects would not necessarily be marked at energies other than those near the threshold for meson production; the lack of their appearance in the observations of Kaplon and Ritson is not in contradiction with the hypothesis of a meson-meson force.[89]

The most recent speculations regarding nuclear forces have been mainly concerned with the pseudoscalar theory with pseudoscalar coupling or else the pseudoscalar theory with pseudovector coupling. These theories are commonly referred to in the abbreviated form $PS(ps)$, $PS(pv)$, the first two letters designating the type of field, the last two—the type of coupling. The $PS(ps)$ and $PS(pv)$ possibilities give similar results in many calculations. This fact has been noted in a limited form by Nelson,[90] then more completely by Bethe[91] and a formal proof has been supplied by Dyson,[92] who showed that the pv coupling interaction Hamiltonian can be made to change aspect by performing a canonical transformation involving simultaneously the nucleon and the meson quantized wave functions. The transformed Hamiltonian contains then terms which imply a δ-function type interaction between nucleons which has been usually thrown away because it appears at first sight not to be called for by experience. Modifications of the δ-function interaction which result on taking nuclear recoil into account spread the interaction through a distance of the order of $\hbar/Mc$ and are connected with the formation of the repulsive core. In addition, the Hamiltonian contains an interaction energy the main part of which is represented by ps coupling. These results apply if the additionally assumed interaction between nucleons is of the ordinary or spin exchange type but not if it contains charge exchange terms. A more thorough discussion and detailed proof are given by Case[93] who supplies the terms which should be added in the second order of the usual interaction constant. One set of terms represents a contribution to the nucleon mass. Another set has the property of leaving the magnetic moments unchanged. Calculations performed in the $PS(ps)$ theory can thus be used as a labor-saving device for some of the calculations on the $Ps(pv)$ theory. The equivalence is by no means complete, however, and both forms of theory have received considerable attention in the literature. The fact that in a number of important cases the equivalence is of little help has been especially noted by Feynman,[94] and the subject has been reopened by Wentzel[95] who approaches the problem by means of Foldy's transformation[96] and finds a difference between the two types of coupling both regarding nuclear forces and meson phenomena such as scattering of mesons by nucleons and meson-pair creation. Renormalization prescriptions have proved feasible for the $PS(ps)$ case only. This theory is preferable, therefore, even though the whole renormalization procedure still depends on operations with an expansion in powers of the interaction constant g which shows no evidence of convergence after renormalization except for the larger distances between nucleons.

In the first formulations of meson theories of nuclear forces the main effect under discussion has been the so-called second-order interaction energy. This energy is obtained by working out the change in the wave function of the whole coupled system to the first order in the inter-

[88] K. Brueckner and K. M. Watson, Phys. Rev. **87**, 621 (1952).

[89] Considerations of Dyson, Bethe, Chew, and coworkers regarding this force on the $PS(ps)$ theory have been reported at the 1952 Rochester Conference and the January, 1953 (Cambridge) meeting of the American Physical Society.

[90] E. C. Nelson, Phys. Rev. **60**, 830 (1941).

[91] H. A. Bethe, private communication to F. J. Dyson.

[92] F. J. Dyson, Phys. Rev. **73**, 929 (1948).

[93] K. M. Case, Phys. Rev. **76**, 14 (1949).

[94] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[95] G. Wentzel, Phys. Rev. **86**, 802 (1952).

[96] L. L. Foldy, Phys. Rev. **84**, 168 (1951).

action constant commonly called g and then calculating the expectation value of the total energy. Any nucleon changes the wave function by terms of order g, producing a field which may be called the self-field of the nucleon. In the calculation of the total energy the self-field of one nucleon gives rise to terms obtained through the application of the interaction energy of a second nucleon and there results a contribution of order g^2. The second stage of this process is describable as the result of the absorption by the second nucleon of a meson that has been produced by the first. The momentum imparted to the meson by the first nucleon when the meson is emitted is transferred to the second nucleon when the meson is absorbed. The two nucleons thus exert a force on each other. The second-order process is conveniently pictured by means of a Feynman diagram[94] as in Fig. 8. Nucleon and meson lines are represented by full and dotted lines, respectively. The diagram represents schematically the course of events in space time. The exchange of momentum can be conveniently pictured by means of Feynman diagrams also for higher order interactions. Thus in Fig. 9 there are shown fourth-order processes in which the momentum exchange takes place twice, each time through the emission and absorption of a meson. The difference between Fig. 9(a) and Fig. 9(b) is only that of relative order in which the mesons are emitted and absorbed.

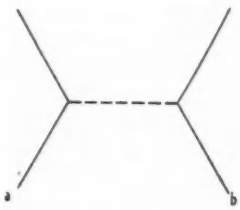

FIG. 8. Feynman diagram for interaction of two nucleons (solid lines) by exchange of a meson (dotted line), the so called second-order interaction energy.

These interactions are of the "fourth-order" type, the coupling parameter entering the expression for the force or energy as g^4. In theoretical calculation it is necessary to deal with contributions to the force by the effects of such exchanges of momenta arising from higher powers of g as well. These correspond to Feynman diagrams with an increasing number of meson lines. Only in the special case of the neutral meson theory in the static approximation is there available a direct elimination procedure enabling one to remove the meson field from the problem by the mathematical device known as the canonical transformation.[97] In practice it has proved necessary to consider contributions from different orders, one at a time.

Several difficulties arise, among them the divergence difficulty. Diagrams in Fig. 10 illustrate situations which can give rise to a divergence. In Fig. 10(a) a meson has been emitted and reabsorbed by one of the nucleons before momentum has been imparted by a meson arriving from the other nucleons. This diagram represents the fact that a nucleon can emit and absorb the emitted mesons all the time, maintaining a kind of an atmosphere of mesons around it. Since the meson field around a point nucleon behaves as $1/r$ at small r there results an infinite contribution to the energy of the field. This energy is rather similar to the infinite energy residing in the electromagnetic field around a point charge. It would have been more satisfactory if theoretical considerations were not concerned with infinities at all. The mere presence of an infinite energy in the field around a point charge does not constitute an insuperable difficulty, however. Methods have been devised[98] for their systematic subtraction. The guiding principle in these procedures is essentially one of symmetry. A simple and somewhat too elementary example of how symmetry can be used is found in the consideration of the force on a point charge in elementary electrostatics. As long as one supposes that the charge is spherically symmetric there is no question concerning the existence of a force on the point charge which might have its origin in the field produced by the charge itself. It is realized, of course, that the total force is zero anyway, since action and reaction are equal opposite according to Newton's third law. Nevertheless, the assumed spherical symmetry also shows that the force is zero. When the charge is set into

[97] G. Wentzel, *Quantentheorie der Wellenfelder* (Franz Deuticke, Vienna, 1943); especially p. 46.

[98] S. Tomonaga, Prog. Theoret. Phys. **1**, 27 (1946); T. Tati and S. Tomonaga, Prog. Theoret. Phys. **3**, 391 (1949); Fukuda, Miyamoto, and Tomonaga, Prog. Theoret. Phys. **4**, 47, 121 (1941); J. Schwinger, Phys. Rev. **74**, 1439 (1948); **75**, 651 (1949); **76**, 790 (1949); R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949); F. J. Dyson, Phys. Rev. **75**, 486, 1736 (1949); **82**, 428 (1951); **83**, 608, 1207 (1951); Proc. Roy. Soc. (London) **A207**, 395 (1951).

motion, however, there appears in the calculations an infinite force which is associated with the presence of accelerations and higher time derivatives of the displacement. The classical researches of Abraham[99] and Lorentz[100] have shown that one can consider this infinite force as the limit of the force on a spherical distribution of charge and introduce in this manner the concept of electromagnetic mass. If one simply calculates the electromagnetic momentum in an

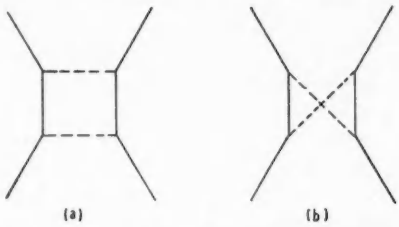

FIG. 9. Feynman diagrams for the two possible fourth-order nucleon-nucleon interactions in which two mesons are exchanged by the interacting nucleons.

arbitrary frame of reference one obtains a dependence on the velocity which is in contradiction with restricted relativity. It has been shown by von Laue[101] and Einstein how this contradiction can be removed through the consistent consideration of the Poincaré stresses which have to exist in order to keep the spherical model of the electron from blowing up under the influence of the electrostatic forces. It is clear, however, that if one simply requires that the total momentum should vary with the velocity of the particle in accordance with the formulas of restricted relativity then one obtains just as much information for practical purposes as though the more satisfactory consideration of compensations in the effects of Poincaré stresses and of the electromagnetic momentum were made. Here again one is helped by a principle of symmetry, *viz.*, the principle of covariance under Lorentz transformations. The value of the electromagnetic mass has no practical significance because one has to introduce the Poincaré stresses whose only function is to save the logic of the structure. The advances in subtraction techniques proceed somewhat similarly to the simpler method in the example just mentioned. Photons or mesons surrounding an electron or nucleon which are schematically represented by the closed loop in Fig. 10(a) contribute an infinite amount to the self-energy but the infinity can be cancelled by introducing infinite changes in the mass of the particle. Computational techniques have been developed for performing this subtraction consistently regarding the relationship of the energy to the momentum. The techniques are in this respect similar to that elaborated on in the example of Poincaré stresses in electrostatics and consist in keeping the calculations in a form in which the covariance features can be enforced at any stage.

In addition to the problem of handling the mass terms it is also necessary to have proper methods for handling other radiative effects. Unifying and simple statements are more difficult to make regarding the procedures. Aside from explicit and elaborate discussions of the calculational recipes, one way of stating them[102] is that the calculations should be made by requiring a symmetry in the momentum space associated with the fermion. Another and probably better way is that of Schwinger's proper-time technique[103] in which special emphasis is laid on the performance of the integration over

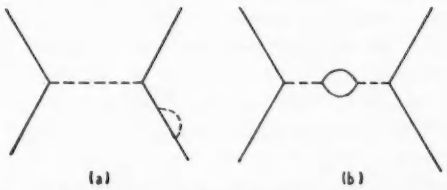

FIG. 10. (a) Diagram for the divergence difficulty arising from the mesic self-energy of the nucleon. (b) Diagram for the divergence difficulty arising from the polarization of the vacuum.

the "proper time" and the enforcement of the covariance requirements at a preceding stage.

In Fig. 10(b) one sees another characteristic phenomenon. The photon or meson ejects a fermion (proton or electron) from a negative energy state producing a positron-electron or proton-antiproton pair. Since the physical vacuum is supposedly filled by electrons and protons

[99] M. Abraham, Ann. Physik **10**, 105 (1903).
[100] H. A. Lorentz, Physik. Z. **2**, 78 (1900).
[101] M. von Laue, *Die Relativatstheorie* (Vieweg and Sohn, Braunschweig, 1921), Band 1.
[102] G. Breit, Science **113**, 103 (1951).
[103] J. Schwinger, Phys. Rev. **82**, 664 (1951).

in negative energy states there result effects modifying electromagnetic action. These are known as vacuum polarization effects. The requirements of covariance regarding Lorentz transformations and gauge invariance make it possible to deal also with these phenomena in a consistent manner. Effects shown in Fig. 11 are

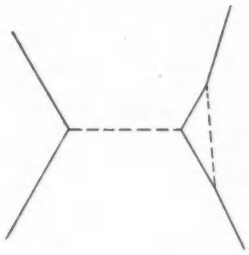

FIG. 11. Diagram for the effect responsible for the Lamb shift and associated phenomena.

responsible for the Lamb shift and associated phenomena which gave the first opportunity to compare the newer methods of calculation with experiment.

The vacuum polarization results also in infinite effects, the situation being similar to that of electromagnetic mass. Both mass and charge have to be changed from the originally assumed values and infinite integrals are encountered in this renormalization process.

For quantum electrodynamics the renormalization procedure can be carried out self-consistently[104] as long as the results are expanded in powers of e. There are reasons for believing that the renormalizable result does not converge but[105] it is satisfactory that up to a point the procedure is self-consistent and agrees with experiment.

The situation is more complicated in meson theory. Extra terms involving additional powers of momenta appear in the calculations and convergence is more difficult to enforce.[94] When it can be enforced there is more ambiguity regarding the answer. Ways of forming renormalizable meson theories have been formulated by Dyson, Matthews,[106] and Salam[107] but in these developments the introduction of δ-type interactions between mesons is considered permissible. Aside from these difficulties of mathematical formulation in essence there are difficulties of carrying through the calculations. The renormalization procedures are defined only with respect to Feynman diagrams, i.e., with respect to an expansion in powers of g. There is no assurance that the series in powers of g converges and it is probable that it does not. From a purely logical viewpoint it appears that there is no meson theory of nuclear forces.

Nevertheless[108] π-mesons are known to appear in high energy collisions between nucleons and they are known to be scattered anomalously by pions. There is a strong suspicion, therefore, that nuclear forces are associated with pions. If the primary interaction process did not consist in the production of real or virtual mesons, the divergence difficulties would be reduced and possibly eliminated. A possible view[41] is to consider the mesons as emitted at a later stage in the process. Then an isolated nucleon would not have to be capable of producing mesons, but their emission could be thought of as a result of the formation of special nucleonic states which arise as a consequence of nucleon-nucleon interactions. This view would eliminate the mesic self-energy of the nucleon, provided the interactions responsible for the formation of the internally activated states were treated in such a way as not to introduce it again through collisions with particles which must be supposed to occupy the negative energy states. The separation of states according to sign of energy can be performed in a relativistically invariant manner so a formulation eliminating the effect of collisions with particles in negative energy states appears to be possible. Such a theory has not yet been developed, however, and it is not known whether one can account for nuclear forces by means of the sensitized state hypothesis. Recent attempts along more standard lines are concerned with the mathematically unsatisfactory expansion in powers of the interaction constant. In the hope that the mathematical inconsistencies will be eventually removed it appears proper to give a brief account of some of the developments. The more hopeful of the recent attempts can be related in part to the remarks of Wick[109] con-

[104] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[105] F. J. Dyson, Phys. Rev. **85**, 631 (1952).
[106] P. T. Matthews, Phil. Mag. **41**, 185 (1950).
[107] A. Salam, Phys. Rev. **82**, 217 (1951).
[108] E. Gardner and C. M. G. Lattes, Science **107**, 270 (1948); Burfening, Gardner, and Lattes, Phys. Rev. **75**, 382 (1949).
[109] G. C. Wick, Nature **142**, 993 (1938).

cerning the expectation that higher orders in g may be expected to give rise to interactions of shorter range. The argument is essentially that virtual states involving the simultaneous presence of several mesons give larger energy changes. Since the rest mass of the meson determines the range through the presence of the rest mass energy μc^2 in the denominator of the expression

$$\hbar c/(\mu c^2)$$

for the range constant, the simultaneous emission of two mesons gives a range constant essentially $\frac{1}{2}$ of that for one meson, being approximately equivalent to the emission of a fictitious meson of mass 2μ. The exponential factor e^{-x} in the Yukawa formula for the potential energy may in fact be qualitatively explained as arising from the spatial exponential decay of the wave function representing one nucleon and one meson; the origin of the exponential decay is similar to that of the approximately exponential decay of an alpha-particle leaking through a Coulomb barrier. An increase in the energy of the virtual state increases the absolute value of the energy deficiency and there is an increase therefore in the strength of the exponential decay. A consideration of the relativistic features of the problem changes the dependence on $1/\sqrt{\mu}$ to $1/\mu$ and the shortening of the range is expected to occur approximately as $1/n$ in the order in which g occurs as g^{2n}. The shape of the nuclear potential energy curve is expected to contain short range contributions arising from the higher order terms in g. This idea has been discussed by Nishijima[110] in connection with the $PS(ps)$ and $PS(pv)$ forms of theory. At the same time Nishijima points out that the matrix γ_5 which enters the ps coupling produces energy changes of order $2Mc^2$ and brings in therefore a characteristic length $\hbar/(2Mc)$ = Compton wavelength of nucleon/(4π). For this reason there occur contributions to the potential energy which vary significantly within these very short distances. Terms varying in this manner are referred to as contact terms. Machida and Nishijima[111] have published results of more concrete calculations together with considerations concerning non-static effects, carrying further views regarding the role of recoil effects brought out by Rosenfeld.[112] They pay special attention to the precession frequencies of the ordinary nuclear spin and the isotopic spin and point out that the distance Rc, at which these frequencies, expressed in energy measure, become comparable with the rest mass of the meson may be regarded as critical in the consideration of the potential. The angular velocities related to the precession frequencies are thus required to satisfy relations $\hbar\Omega<\mu c^2$, $\hbar\omega<\mu c^2$ for $r>R_c$. Here Ω, ω are the precession frequencies for ordinary and isotopic spin, respectively. For potentials involving steep variations at small distances they estimate $R_c \sim \hbar/(2\mu c)$, for more gently varying potentials $R_c \sim \hbar/(4\mu c)$. These speculations doubtless have a bearing on the determination of the distance at which the potentials have to be modified through the introduction of velocity-dependent terms. With the aid of the considerations of Nambu[113] regarding formal aspects of the two-body problem, concerned with interactions arising as a result of meson exchange represented in Fig. 9, Taketani, Machida, and Ônuma[114] have worked out the fourth-order potential on the symmetrical pseudoscalar theory employing both pv and ps coupling. In addition they carried through numerical applications to the low energy phenomena. Their results are as follows. Second-order potentials on the pseudoscalar theory,

$$V^{(2)} = -\kappa(g^2/4\pi)\{e^{-x}/x + (1+3/x+3/x^2)(e^{-x}/x)S_{12}\}(x>x_0),$$

and fourth-order potentials with (pv) and (ps) coupling,

$$V_{pv}{}^{(4)} = \kappa(g^2/4\pi)^2[(\tau^{(1)}\tau^{(2)})U_\tau(x) + (\sigma^{(1)}\sigma^{(2)})U_\sigma(x)+S_{12}U_T(x)],$$

$$V_{ps}{}^{(4)} = \kappa(f^2/4\pi)^2(3/8\pi)(\mu/M)^2K_1(2x)/x^2,$$

[110] K. Nishijima, Prog. Theoret. Phys. 6, 815 (1951).

[111] S. Machida and K. Nishijima, Prog. Theoret. Phys. 7, 57 (1952).

[112] L. Rosenfeld, *Nuclear Forces* (North Holland Publishing Company, Amsterdam, 1948), p. 324. The effect of recoils on nuclear force has been discussed formally by Araki, Prog. Theoret. Phys. 4, 193 (1949); 6, 379, 673 (1951); Phys. Rev. 75, 1101 (1949).

[113] Y. Nambu, Prog. Theoret. Phys. 5, 614 (1950).

[114] Taketani, Machida and Ônuma, Prog. Theoret. Phys. 6, 638 (1951); Machida, Ônuma, and Taketani, Prog. Theoret. Phys. 6, 904 (1951); Taketani, Machida, and Ônuma. Prog. Theoret. Phys. 7, 45 (1952).

where

$$U_\sigma=(8/\pi)\{(3/x^2)K_0(2x)+[(2/x^2)+(3/x^4)]K_1(2x)\},$$

$$U_\tau=-(8/\pi)\{[(1/x)+(23/4x^3)]K_0(2x)+[(2/x^2)+(23/4x^4)K_1(2x)]\},$$

$$U_T=-(8/\pi)\{(3/x^3)K_0(2x)+[(1/x^2)+(15/4x^4)]K_1(2x)\}.$$

Here subscripts σ, τ, on the U indicate the occurrence of the ordinary and of isotopic spins in the coefficients of U. The subscript T similarly indicates the occurrence of the tensor force combination S_{12}. The quantities K_0, K_1 are modified Bessel functions defined in terms of the Hankel functions by

$$K_\nu(x)=(\pi i/2)e^{-\pi i\nu/2}H^{(1)}{}_\nu(ix)\sim(\pi/2x)^{\frac{1}{2}}e^{-x}.$$

The remaining quantities requiring explanation are

$$\kappa=\mu c/\hbar, \quad x=\kappa r.$$

Taketani *et al.*, consider $V^{(2)}$ as being a sum of contributions arising from central and from tensor forces,

$$V^{(2)}=V_c+V_TS_{12}$$

and assume that

$$V_c=\infty, \quad V_T=\infty \quad (x<x_0) \quad \text{(hard core)},$$

or

$$V_c=0, \quad V_T=0 \quad (x<x_0) \quad \text{(zero cutoff)}.$$

The assumption of a hard core is partly suggested by the paper of Nishijima quoted above and partly by the work of Jastrow.[69] In the exterior region $x>x_0$ the $PS(ps)$ contribution to $V^{(2)}$ is negligible.

The second-order potential with a repulsive core gave results as in Table V. The experimental $Q=2.74\times10^{-27}$ cm^2 favors a value of g^2 close to that corresponding to the last row. Such a value of g^2 is found to contradict the experimental value of the singlet scattering length, however.

TABLE V. Second-order $PS(pv)$ potential outside, hard core inside, according to Taketani, Machida, and Ônuma (see reference 114).

$g^2/4\pi$	x_0	Q (10^{-27} cm^2)	P_D (percent)
0.025	0.122	1.07	4.2
0.050	0.228	1.877	6.3
0.075	0.387	2.542	7.4
0.084*	0.458	2.766	7.54

* The last row in the table was obtained by extrapolation.

For the combination of fourth- and second-order potentials they find results as in Table VI.

TABLE VI. Singlet $n-p$ scattering according to Taketani *et al.* (see reference 114).

$g^2/4\pi$	${}^1r$	x_0	${}^1r$	x_0
0.075	1.925	0.589	2.104	0.328
0.090	2.457	0.665	2.585	0.384

The quantity ${}^1r$ is the singlet effective range the experimental value of which is $\sim2.7\times10^{-13}$ cm. According to Table VI one can fit the effective range by means of the hard core model and a large $g^2/4\pi$. In Table VII there is shown a fit to the deuteron constants.

TABLE VII. Deuteron ground state and triplet $n-p$ scattering zero cut-off, fourth-order theory, according to Taketani *et al.* (see reference 114).

$g^2/4\pi$	x_0	Q (10^{-27} cm^2)	${}^3r$ (10^{-13} cm)	P_D (percent)
0.05	0.16	1.85	1.325	7.42
0.08	0.232	2.37	1.582	8.65
0.10	0.303	3.25	1.75	11.3

It is noteworthy that a fit to the deuteron ground state with a hard core model proved impossible. Although the workers point to a possible connection with Jastrow's fits to $p-p$ scattering in which a hard core is used this connection is very tenuous, since in even singlet states the potential used by Taketani *et al.*, is attractive.

Related investigations have been carried out by Lepore[115] and independently by Lévy[116] employing different methods of calculation. The results are closely similar and are concerned with the $PS(ps)$ theory. Lévy's result is

$$V_{ps}{}^{(2)}=(f^2/4\pi)(\mu/2M)^2(\tau_1\tau_2)\{(\sigma_1\sigma_2)+(1+3/x+3/x^2)S_{12}\}e^{-x}/r,$$

$$V_{ps}{}^{(4)}=-3(f^2/4\pi)^2(\mu/2M)^2(1/\mu r^2)\times\{2/\pi K_1(2x)+\mu/2M[(2/\pi)K_0(x)]^2\}.$$

[115] J. V. Lepore, Bull. Am. Phys. Soc. **27**, No. 3, 33 (1952); R. Jastrow, Bull. Am. Phys. Soc. **27**, No. 3, 33 (1952).

[116] M. M. Lévy, Phys. Rev. **86**, 806 (1952); Compt. rend. **234**, 815, 922, 1255, 1671, 1744 (1952).

The first term of $V_{ps}^{(4)}$ agrees in form with Taketani *et al.* There was apparently an omission in their work of the effect on $V^{(4)}$ of states in which only one meson is present in intermediate states. Since the numerical results are concerned with the $PS(pv)$ case they are presumably unaffected by this circumstance. The $PS(ps)$ theory is, however, the one of greater interest since there is no consistent renormalization procedure for the $PS(pv)$ case.[116a]

By employing $f^2/4\pi=9.7$ and a hard core with $x_0=0.38$, an approximate fit to the deuteron ground state and low energy scattering in the shape independent approximation was obtained by choosing f^2, x_0 so as to fit the deuteron binding energy and the $n-p$ scattering length and then calculating by means of f^2 and x_0 the quantities

$$^1r=\text{Singlet effective range}=2.47\times10^{-13}\text{ cm}$$

$$^3r=\text{Triplet effective range}=1.64\times10^{-13}\text{ cm}$$

$$Q=2.08\times10^{-27}\text{ cm}^2.$$

The agreement with experiment is very good and this explanation appears to be a very hopeful one. Although Q is too low it may become improved through the inclusion of higher order corrections. From $p-p$ scattering the value of ^1r is

$$^1r=2\int_0^\infty[(u^s)^2-u^2]dr=2.88\times10^{-13}f^{(1)}\text{ cm}.$$

For the "best linear" fits of YSHBB $f^{(1)}=0.917$, $^1r=2.64\times10^{-13}$ cm; for the "best quadratic," $f^{(1)}=0.956$, $^1r=2.76\times10^{-13}$ cm. The "best meson" fit of YSHBB gives practically the same result as the "best quadratic" fit. The ratios of the empirical to the theoretical values are 1.12 for the best quadratic fit, 1.08 for the linear. The difference from unity has the appearance of being significant. Since the difference between 1.12 and 1.08 is $\frac{1}{2}$ of the difference between 1.08 and unity, it would be improper to overemphasize this discrepancy in the present preliminary stage of this very promising development. The theoretical $f^{(1)}$ is smaller than the empirical. A progressive increase of $p-p$ attraction gives a smaller apparent range parameter and tends to make the apparent (experimental) $f^{(1)}$ smaller. The direction of the discrepancy is thus such as though an additional repulsion sets in as the proton energy is increased. In the above comparison two obvious effects are omitted: (1) The effect on $f^{(1)}$ of changes in u^s and u caused by the Coulomb field. (2) The modification in the meson potential caused by the presence of an extra charge in the $p-p$ system as compared with the $n-p$ system.

The fit to experiment appears to be supported by a recent calculation of the magnetic moment of the neutron made by Fried[117] in connection with the $e-n$ interaction. Fried finds the constant $f^2/4\pi=7.3$ on the $PS(ps)$ theory. His calculation takes into account only terms in the second power of f^2 and a perfect fit can hardly be expected.

Considerations regarding the magnitude of the coupling constant have to be used with considerable caution. Matthews and Salam[118] discuss methods of calculation not depending on the smallness of the coupling constant which they compare with a renormalized weak coupling expansion for a $PS(ps)$ theory. They conclude that in a consideration of the self-energy problem, even for $f^2/4\pi\sim10$, a few terms of the expansion should give a reliable representation of the meson cloud. However, calculations of Koba, Kotani, and Nakai[119] of the cross section for photomeson production for the $PS(ps)$ theory (among others) indicate that for $f^2/4\pi\sim1$ the fourth-order corrections are not much smaller, for γ-rays of 330-Mev energy, than the second-order terms. Progress may be expected from the employment of considerations of an over-all type regarding the meson cloud around a nucleon. For very high energy phenomena Fermi[120] proposed a statistical treatment and the problem of magnetic moments has also been discussed by Umezawa, Takahashi, and Kamefuchi,[121] who reproduce the results of Pauli and Dancoff[122] obtained by more formal methods. More recently

[116a] H. A. Bethe, Rochester Conference on High Energy Physics (1951).

[117] B. D. Fried, Phys. Rev. **86**, 434 (1952).

[118] P. T. Matthews and A. Salam, Phys. Rev. **86**, 715 (1952).

[119] Koba, Kotani, and Nakai, Prog. Theoret. Phys. **6**, 849 (1951).

[120] E. Fermi, Prog. Theoret. Phys. **5**, 570 (1950).

[121] Umezawa, Takahashi, and Kamefuchi, Phys. Rev. **85**, 505 (1952).

[122] W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942).

Sachs has considered the problem more concretely. He employs a nonrelativistic treatment of meson contributions to the magnetic moments and the results are therefore not comparable with those of calculations along more standard lines. He arrives at the conclusion that one cannot fit the magnetic moments of the neutron and proton by supposing that the meson cloud contains one pion on the average. An excellent fit is obtained by him, however, by supposing that the most probable pion containing state has two pions present simultaneously. The probability of such a state is estimated by Sachs as ~9 percent. The general condition of the meson cloud arrived at in this manner does not support the nuclear force $PS(ps)$ model and suggests the presence of a weak nonlinear coupling[123] of mesons with nucleons.

While there are objections to the $PS(ps)$ meson theory it appears, nevertheless, to be one of the more hopeful attempts at the solution of the nuclear two-body problem. In particular, one can hope that a natural reconciliation with the mass of the meson will be attained. A feature tending in this direction is the increase in the nucleon-nucleon attraction caused by $V^{(4)}$ in the region just outside the repulsive core. In comparison with the old-fashioned Yukawa potential one obtains therefore a steeper decrease of the attraction, as though the meson mass were increased. It is premature to claim success in this connection before the velocity dependent features of the interaction are understood.[42,43] The energy rate of change of phase shift is affected by velocity dependence.

A special and apparently vital feature of Lévy's consideration is the inclusion of effects of nucleon recoils in the discussion of the "contact type" terms. These are present in the older nonrelativistic considerations in the form of terms proportional to $\delta(r)$. The inclusion of nuclear recoil spreads this localized interaction in such a way as to form the repulsive core.

In evaluating the agreement with experiment obtained by Lévy it may be borne in mind that the radius of the core was an adjustable parameter. This procedure was used because the theory is presumably not good enough to give details such as the shape of the steeply increasing part of the potential energy curve. Reasonable as such arguments are, the necessity of using them detracts somewhat from one's confidence in the theory.

Note added in proof: Considerations of G. Wentzel and of I. W. Ruderman discussed at the 1953 Rochester Conference on High Energy Physics throw doubt on Lévy's claim that $V^{(4)}$ is the last important term for the static nuclear potential. Quantitative aspects of Lévy's reconciliation of scattering data with the observed mass of the meson are very questionable therefore.

General Features

Symmetry of Hamiltonian

It has become customary to assume that the nuclear Hamiltonian is symmetric, i.e., that aside from Coulomb effects and inequalities in interactions resulting from differences in magnetic moments the forces between all nucleons depend only on their distances, spin orientations, etc., but not on whether they are neutrons or protons. The early evidence[124] has never been quantitatively conclusive but suggested an approximate relationship. The equality of $p-p$ and $n-n$ forces was first suggested by the possibility of accounting for Coulomb energies of mirror nuclei. To this evidence one can now add additional more detailed calculations based on better experimental material with extensions to excited states.[125] There is also evidence arising from the behavior of the $n-n$ system in free space as distinguished from its behavior as part of a nucleus. This is of two kinds. The Los Alamos Scientific Laboratory investigators[126] studied the reaction

$$H^3+H^3\rightarrow He^4+2n.$$

Two peaks in the curves representing the number

[123] L. I. Schiff, Phys. Rev. **84**, 1, 10 (1951); R. J. Glauber, Phys. Rev. **84**, 395 (1951); B. J. Malenka, Phys. Rev. **85**, 686 (1952).

[124] E. Feenberg and J. Knipp, Phys. Rev. **48**, 906 (1935); E. Feenberg and S. Share, Phys. Rev. **50**, 215 (1936); S. Share, Phys. Rev. **50**, 488 (1936); Breit, Condon, and Present, Phys. Rev. **50**, 825 (1946); G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936); E. Wigner, Phys. Rev. **51**, 106 (1937).

[125] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950); Brown, Snyder, Fowler, and C. C. Lauritsen, Phys. Rev. **82**, 159 (1951); D. R. Inglis, Phys. Rev. **82**, 181 (1951).

[126] Los Alamos Laboratory, Phys. Rev. **79**. 238 (A) (1950).

of alpha-particles against energy are observed. The higher energy peak corresponds approximately to the energy which would result if the two neutrons moved as a unit. An approximate analysis of the results suggests that the binding force between the neutrons corresponds to a virtual state of the dineutron of a few hundred kilovolts. This evidence is not believed to be very strong and surely is not conclusive. It is mentioned because future study of this reaction might give improved information. Another and rather convincing demonstration of the $n-n$ force in free space is found in work started by Panofsky, Aamodt, and Hadley[127] and the realization by Tamor and Marshak[128] and independently by Brueckner, Serber, and Watson[129] that the process

$$\pi^- + D \rightarrow 2n + \gamma$$

can be used to determine the $n-n$ interaction in the ^1S state. Watson and Stuart[130] have calculated the theoretical gamma-ray spectra resulting on the reasonable view that the primary process is

$$\pi^- + p \rightarrow n + \gamma.$$

The most recent and complete work appears to be that of Phillips[131] under the guidance of W. Panofsky. The result is a scattering length

$$^1a_{nn} \cong 15.7 \times 10^{-13} \text{ cm}.$$

The value is not supposed to be accurate, all values in the range

$$8.7 \times 10^{-13} \text{ cm} < {}^1a_{nn} < \infty$$

being permissible. On the basis of this evidence there is no objection to considering the $n-n$ and $n-p$ interactions as being equal. The comparison would have a more convincing appearance in terms of the reciprocal of ^1a since then the limit ∞ would become 0, corresponding to a virtual state of the $n-n$ system at zero energy.

Among the experiments which give evidence regarding the symmetry of the nuclear Hamiltonian by means of work in "classical nuclear physics" one should mention the results obtained by Latter and Latter[132] concerned with a phase shift analysis of neutron-deuteron scattering. They find that a simultaneous phase shift analysis of $n-d$ and $p-d$ angular cross sections is consistent with the symmetry of the nuclear Hamiltonian even though their theoretical phase shifts are not in agreement with calculations of Buckingham and Massey. The work of McIntosh, Gluckstern, and Sack[133] on $p-T$ scattering and the $n-\text{He}^3$ reaction is somewhat similar in that according to it there is no obvious contradiction between experiment and the assumed symmetry of the Hamiltonian. Additional evidence is supplied by the work of Pease and Feshbach,[78] who calculated the binding energy of the triton using Yukawa wells with central and tensor parts on the assumption of charge independence of the forces. Potential constants were fixed by low energy two-body data except for the tensor range which was determined by the experimental binding energy of the triton. Equivalent $n-p$ triplet ranges computed in this manner showed excellent agreement with experiment. However, the Coulomb energy is about 25 percent high. The work of Thomas[134] indicates that term systems in nuclei are essentially unchanged as one changes a neutron into a proton although they are affected, of course, by the operation of the Pauli exclusion principle. One cannot claim decided quantitative success in these comparisons but the evidence is strong regarding the qualitative aspects.

A question of long-standing interest is the degree of the exactness which may reasonably be required of the symmetry of the nuclear Hamiltonian. An attempt to refine the comparison of the $n-p$ and $p-p$ interactions in the ^1S

[127] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951). Aamodt, Panofsky, and Phillips, Phys. Rev. **83**, 1057 (1951). Cf. also, Cartwright, Richman, *et al.*, Phys. Rev. **81**, 652 (1951).

[128] S. Tamor and R. Marshak, Phys. Rev. **80**, 766 (1950).

[129] Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951).

[130] K. Watson and R. Stuart, Phys. Rev. **82**, 738 (1951).

[131] R. H. Phillips, UCRL 1845, dissertation, University of California.

[132] A. L. Latter and R. Latter, Phys. Rev. **86**, 727 (1952). It may be mentioned in this connection that unpublished work of Christian and Gammel indicates the possibility that $n-p$ and $n-d$ scattering may be caused mainly by the $n-p$ interaction. While these calculations are approximate only, the possibility that these cases are insensitive to differences in the $n-n$ and $p-p$ forces is doubtless present. An earlier discussion of this matter was given by G. Breit, Phys. Rev. **80**, 1110 (1950).

[133] McIntosh, Gluckstern, and Sack, Phys. Rev. **88**, 752 (1952).

[134] R. G. Thomas, Phys. Rev. **81**, 148; **84**, 1061 (1951); J. B. Ehrman, Phys. Rev. **81**, 412 (1951).

state has been made by Schwinger[135] but even this attempt at precision may be regarded as subject to future changes and refinements on account of the changing ideas concerning the shape of the nuclear potential well and the arbitrariness of Hamiltonians used. It should also be mentioned that in Schwinger's comparison a small outstanding difference was left unexplained with the hope that it would eventually be clarified as a result of the differences in the behavior of mesons for the two systems caused by the presence of an extra charge in the proton-proton system. With the advent of new material in the field of "meson physics" it appears probable that a deeper insight in the cause underlying the symmetry of the nuclear Hamiltonian will be obtained. Experiments with mesons are expected by many to yield information regarding a symmetry in the interactions of the three different kinds of π-mesons with nucleons. The meson theory of nuclear forces has been formulated with symmetry in the isotopic spin space by Møller and Rosenfeld. Heitler[136] has shown that such a symmetry has also consequences for cross sections in meson physics which can be subjected to experimental test. Watson and Brueckner[137] and Watson[138] have made contributions to the theory of such symmetries, and Messiah[139] as well as Luttinger[140] have drawn attention to relations between cross sections which can be used in order to test the assumption of the symmetry of pion-nucleon interaction in isotopic spin space. Some of these relations, such as Eq. (1) of Luttinger, are somewhat similar to the connections between intensities of light polarized parallel and perpendicular to the magnetic field in the Zeeman effect. It is to be hoped that developments along these lines will yield information of sufficiently precise character to enable a decision to be made regarding the origin of the approximate symmetry of the nuclear Hamiltonian. It should be mentioned, however, that on the pseudoscalar theory with pseudoscalar coupling there is expected an interaction effect with the electric field which may affect the direct transfer of the fundamental symmetry of the meson-nucleon interaction to the probably more secondary symmetry of the nuclear Hamiltonian.

[135] J. Schwinger, Phys. Rev. **78**, 135 (1950); Hoisington, Share, and Breit, Phys. Rev. **56**, 884 (1939).
[136] W. Heitler, Proc. Roy. Irish Acad. **51**, 33 (1946).
[137] K. M. Watson and K. K. Brueckner, Phys. Rev. **83**, 1 (1951).
[138] K. M. Watson, Phys. Rev. **85**, 852 (1952).
[139] A. M. L. Messiah, Phys. Rev. **85**, 430 (1952).
[140] J. M. Luttinger, Phys. Rev. **86**, 571 (1952).

Composite Character of the Nucleon

Qualitative evidence which makes it unreasonable to attack the problem of nuclear forces exclusively from the viewpoint of static potentials will be briefly reviewed in this section. Such evidence is generically referred to here as describing the composite character of nucleons. In addition to the fact that π-mesons can be produced by photodisintegration of the proton, mention should be made of evidence arising from the high energy photodisintegration of the deuteron. In the early development of nuclear physics it was hoped that some information regarding the range of nuclear forces could be derived from a study of this phenomenon. This view still holds, but it has become clearer that the phenomenon is complicated by a modification which is not explicable in a natural manner by means of static forces. The cross section for photodisintegration is larger than calculated by means of phenomenologic potentials at energies of about 160 Mev as demonstrated by the investigations of Kikuchi,[141] Benedict and Woodward,[142] Gilbert and Rose,[143] and Littauer and Keck.[144] According to this work the ratio of the observed cross section to that calculated from phenomenologic potentials is about two or three. A theoretical calculation by Bruno and Depken[145] pictures the high energy process as occurring through the emission of a π-meson by one nucleon followed by an absorption of a photon by the meson and then followed by an absorption of the meson by the other nucleon. This work is related to that of Austern.[146]

The investigation of the question of additivity of nuclear magnetic moments is also promising to throw light on the degree to which nuclei may

[141] S. Kikuchi, Phys. Rev. **85**, 1062 (1952).
[142] T. S. Benedict and W. M. Woodward, Phys. Rev. **85**, 924 (1952).
[143] W. S. Gilbert and J. W. Rose, Phys. Rev. **85**, 766 (1952).
[144] R. Littauer and J. Keck, Phys. Rev. **86**, 1051 (1952).
[145] B. Bruno and S. Depken, Phys. Rev. **86**, 1054 (1952).
[146] N. Austern, Bull. Am. Phys. Soc. **27**, 32 (1952).

be regarded as composite. Work of Sachs,[147] Avery and Adams,[148] Sachs and Austern,[149] and Osborn and Foldy[150] has shown that nuclear moments are in all probability not additive. A recent investigation of a related question has been made by Villars[151] in connection with possible exchange currents in the deuteron. He shows that in the nonadiabatic approximation one must modify the conclusion that the deuteron is not affected by electromagnetic exchange moment effects.

The question of additivity of nuclear magnetic moments requires for its answer a comparison of theory with experiment which cannot be made without the introduction of relativistic corrections to the magnetic moments of nucleons. Such corrections have been known for some time to be of the order of W/mc^2 for electrons.[152] It has been observed by Margenau[153] that the correction for nucleons may be sufficiently large to warrant consideration and he worked out a formula applicable to central fields and expressing the result in terms of T/Mc^2, where T is the nonrelativistic expectation value of the kinetic energy. Only the Dirac part of the magnetic moment, owing its origin to the proton charge, was calculated. Caldirola[154] called attention to the fact that the intrinsic (Pauli) part of the moment requires a different treatment. His results, judging by his applications, appear to have been distorted by a misunderstanding regarding the sign. Breit[155] calculated the intrinsic moment correction, showing that within the limitations of point nucleons the same results may be applied in many-particle problems. In the same note the modification required for the scalar field is calculated for the one-particle model. While the papers quoted give a reasonably complete picture for the intrinsic moment part, the corrections for the Dirac part of the moment are far less certain for the problem of H^2, because two particles are involved and a direct transfer of central field one-body results cannot be justified. The two-body problem has been treated by Sachs,[156] Breit and Bloch,[156a] and by Adams.[156b] Related considerations have been made by Primakoff.[156c] Sachs calculates relativistic corrections for the vector and scalar cases, employing approximate solutions (static approximation) for the meson fields. The Hamiltonians used are not covariant to the order v^2/c^2, while terms of that order are known to matter. In the work of Breit and Bloch[156a] the Hamiltonians are consistently covariant but there is an error in the calculation as noted by Adams.[156b] The matter has been gone over recently by Breit and Thaler.[156d] The newer work explains the results obtained for the two-body covariant Hamiltonians in terms of one-body effects and extends the results to the pseudoscalar field with pseudoscalar coupling. The explanation in terms of one-body effects is essential because even though the Hamiltonians are covariant they are at best approximations and cannot be good to order v^2/c^2 in the calculation of all quantities since they were intended for the calculation of the energy. It is probable that all but the pseudoscalar case will be of purely academic interest in the future, the pseudoscalar nature of the π-meson having been established by later work. There is a close interplay between the assumptions in the theory of nuclear forces and the magnitude of the relativistic corrections which makes it difficult to attack the problem in parts. There is also an additional effect caused by the shielding action of the proton spin current on the neutron which is of the same order of magnitude as the relativistic corrections. The ideal situation would have been to determine the relativistic corrections, to make corrections for nonadditivity, to determine the proportion of the ^3D state and to test this proportion against predictions of the nuclear force theory. The actual problem has resisted such a well-ordered attack. The only simple feature which has shown itself is that for the pseudoscalar interaction with

[147] R. G. Sachs, Phys. Rev. **74**, 433 (1948).
[148] R. Avery and E. N. Adams, Phys. Rev. **75**, 1106 (1949).
[149] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).
[150] R. K. Osborn and L. L. Foldy, Phys. Rev. **79**, 795 (1950).
[151] F. Villars, Phys. Rev. **86**, 477 (1952).
[152] G. Breit, Nature **122**, 649 (1928).
[153] H. Margenau, Phys. Rev. **57**, 383 (1940).
[154] P. Caldirola, Phys. Rev. **69**, 567 (1946).
[155] G. Breit, Phys. Rev. **71**, 400 (1947).
[156] R. G. Sachs, Phys. Rev. **72**, 91 (1947).
[156a] G. Breit and I. Bloch, Phys. Rev. **72**, 135 (1947).
[156b] E. N. Adams II, Phys. Rev. **81**, 1 (1951); see footnote 20.
[156c] H. Primakoff, Phys. Rev. **72**, 118 (1947).
[156d] G. Breit and R. M. Thaler, Phys. Rev. **88**, 1214 (1952).

pseudoscalar coupling the exact formula for a particle in a definite energy state has the same form as for a free particle. In principle, the answer is simply expressed in terms of the energy spectrum of the proton while it is exposed to the field of the neutron. This spectrum is not known, however, and only speculative approximations to the answer are available.

Proton Isobars

The strong coupling form of meson theory[157] has yielded a formal result regarding the formation of semistable states of nucleons which are referred to as proton or neutron isobars. In the old form of meson theory these states corresponded to definite values of the total angular momentum of the system composed of the bare nucleon and the bound mesons. Various arguments summarized by Wentzel[157] have shown the inapplicability of the strong coupling meson theory. Nevertheless, the general possibility of a virtual state of the composite system formed by the bare nucleon and the surrounding meson cloud has remained an attractive subject for speculation. It has been used[33,42] in an attempt to explain the difference between the observed mass of the π-meson and the apparent mass obtained by fitting the Yukawa potential to $p-p$ scattering experiments. This speculation may be profoundly affected by the changes in shape of the nuclear potential which are expected on the basis of the Lévy[116] and Lepore[115] calculations by means of the $PS(ps)$ meson theories. Nevertheless, if an excited state of a nucleon exists at an energy comparable with that of the mass of the π-meson, one may expect a velocity dependence of the effective depth of the potential energy, and this will have to be eliminated in the comparison of experiment with static potentials derivable from meson theories.

The scattering of π-mesons by hydrogen contains related information. An explanation of earlier experiments has been proposed by Brueckner.[158] It was first reported at the Rochester conference on high energy phenomena and was based on measurements by Steinberger *et al.*, and by Fermi *et al.* In Brueckner's explanation there were present two intermediate states which were held responsible for contributions to the scattering cross section, but the principal conclusion was that the state with isotopic and ordinary spin $\frac{3}{2}$ was probably the main one concerned. The result to be explained was the larger value of the scattering of π^+ as compared with that of π^-. In the region where comparison is possible this ratio was approximately 2.5. At this conference it was pointed out by Fermi that the approximate relationship suggested by experiment does not require invoking simultaneously isobaric states with $\tau=\frac{3}{2}$ and $\frac{1}{2}$ but that $\tau=\frac{3}{2}$ suffices. The simple $\tau=\frac{3}{2}$ view leads to a ratio of 3. Brueckner's considerations made use of a formal way of discussing resonance employing the formulas of Wigner and Eisenbud. It is not too clear from the experimental evidence that in this case one is dealing with a well-pronounced resonance. In fact, the cross section for π^- scattering reaches a rather flat plateau at about 200 Mev.

The subject has been investigated again by Fermi, Anderson, and Nagle[159] and the results have been partly published.[160] According to the newer results it does not suffice to explain the data on the assumption of p scattering alone and the whole phenomenon appears to be appreciably more complicated than has been thought at first. It is especially noteworthy that s wave phase shifts are required and that the phase shift analysis performed by the Chicago group involved six phase shifts. The pion-nucleon interaction appears to be strongest in the $P_{\frac{3}{2}}$ states with isotopic spin $\frac{3}{2}$, but the $P_{\frac{3}{2}}$ state of isotopic spin $\frac{1}{2}$ gives a comparable contribution. The pronounced backward scattering is explained through the introduction of an s state scattering anomaly with isotopic spin $\frac{3}{2}$. The phenomenon suggests transitions through intermediate states. It is not clear that the intermediate states have an especially long life. Wentzel[95] discusses meson-nucleon scattering semiqualitatively on the $PS(ps)$ theory using Foldy's transformation and a high energy cutoff and finds that in the incomplete treatment given the cross section does not depend on the isotopic spin in S states, in disagreement with experimental results of Fermi,

[157] G. Wentzel, Revs. Modern Phys. 19, 1 (1947).
[158] K. A. Brueckner, Phys. Rev. 86, 626 (1952).
[159] Fermi, Anderson, and Nagle (preprint).
[160] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. 86, 793 (1952); H. L. Anderson and E. Fermi, Phys. Rev. 86, 794 (1952).

Anderson, and Nagle. He suggests that a crucial test of the $PS(ps)$ theory will be in its ability in a complete treatment to account for the experimentally observed pion-nucleon scattering. The arguments also cast doubt on the usefulness of static approximations to the theory and on earlier conclusions based on weak coupling expansions that higher order corrections increase the two nucleon interactions.[161] Brueckner's work on the isobaric state has been preceded by other considerations made by himself concerning the ratio of reaction cross sections for

$$\sigma[P(\gamma, \pi^0)P]/\sigma[P(\gamma, \pi^+)N]=2/1$$

and is related to the work of Watson and Brueckner[127] on π-meson production in nucleon-nucleon collisions. In the same connection the considerations of Bethe and Austern[162] regarding the angular distribution of π^+ mesons in $n-p$ collisions are likely to lead to further developments.

Shell Structure

Rapid advances in the systematics of nuclear energy levels are being made on the basis of explanations for the existence of magic numbers which have been advanced by M. G. Mayer[163] and independently by Jensen, Haxel, and Suess.[164] It would be out of place to discuss this subject in detail. The general features of the shell structure theory present questions which have to be mentioned, however, in connection with any discussion of nuclear forces. Shell structure theory makes two assumptions: The first is that there exists a decided preference for $j-j$ coupling especially in the heavier nuclei. The second is that the spin-orbit interaction is large and gives rise to an inverted order of the levels, i.e., that for a single nucleon in a central field the level with highest j lies lowest. The evidence for the first assumption is stronger than for the second. In fact it is not clear that the second assumption is necessary because there appear to be possibilities of employing the tensor force successfully in this connection. It has been shown by Feingold and Wigner[71] that the tensor force might be responsible for some of the effects usually attributed to the spin-orbit interaction. The effect of the tensor force in Li^6 and Li^7 has been treated by Feingold[165] in considerable detail in his Princeton dissertation following the general idea of the above reference. Feingold finds that the effect of configuration interaction is about 10 times that of the lowest configuration alone, and on taking this into account by a variational calculation he produces the inverted order of the $^2P_{\frac{1}{2}}-{}^2P_{\frac{3}{2}}$ states of Li^7 and the $^1S_0-{}^3S_1$ states of Li^6 for which there is experimental evidence. The magnitude of the splitting is in good agreement with experiment for Li^7, and roughly 50 percent of the splitting is reproduced for Li^6. The observed moments, particularly the sign of the Li^7 quadrupole moment, are not reproduced theoretically, but Feingold suggests that more careful perturbation calculations with the higher states may help to clear up these points.

The evidence accumulated by Mayer, Goldhaber, Way, and others indicates the validity of the $j-j$ coupling view. The calculations which justify considering the lowest level of a system composed of an odd number of nucleons as having a value of the resultant j equal to that of the individual j's are based on the picture of a δ-function type of interaction. It is remarkable that the consequences of such an interaction agree with experiment in a number of cases. A δ-function type of potential is rather different from the potentials suggested by work on the two-nucleon system. In some cases the same results can be obtained with another shape dependence of the potential.

Phenomena treated by means of shell structure theory are not necessarily simply related to the problem of forces between two nucleons. Thus the work of Schiff[128] on nonlinear meson theories indicates the existence of a shielding action of the effect of one nucleon on another caused by other nucleons in between. Any con-

[161] K. M. Watson and J. V. Lepore, Phys. Rev. **76**, 1167 (1949).

[162] H. A. Bethe and N. Austern, Phys. Rev. **86**, 121 (1952). In the same connection it is of interest to quote a note by Matsuyama and Miazawa on the "Interpretation of the Meso-Disintegration of the Deuteron in Terms of the Strong Coupling Theory," which has been kindly supplied to the authors before publication. The calculated angular distribution is in satisfactory agreement with observations, but the strong coupling calculations are subject to uncertainties.

[163] M. G. Mayer, Phys. Rev. **74**, 235 (1948); **78**, 16 (1950); **78**, 22 (1950).

[164] Jensen, Haxel, and Suess, Naturwiss. **35**, 376 (1948); **36**, 153, 155 (1949).

[165] A. M. Feingold, thesis, Princeton (1952).

sideration which involves "many-body forces" can make the transition from the two-nucleon potential to effective potentials in a heavy nucleus rather difficult. It may be recalled that in the initial development of ideas concerning shell structure it has been thought probable that large contributions to spin-orbit interactions take place at the nuclear surface. Such ideas would have to be modified if the nonlinear meson theory or the many-body force theory effects should turn out to be pronounced. The additional body of information which has a strong bearing on this phase of the problem is that of the passage of high energy particles through matter investigated by the Berkeley school and analyzed by R. Serber[166] and collaborators. In these phenomena one is dealing with a somewhat simplified condition on account of the high energy of the particles and a successful purely phenomenological approach has proved possible without making specific and binding assumptions regarding the nature of nuclear forces.

Early hopes of rapid progress in the understanding of nuclear forces through energy level studies have not been fulfilled so far. There is perhaps no reason for history to repeat itself in favoring "spectroscopy" as the principal tool in the advance of a branch of physics. To a degree one can draw a parallel between the roles of molecular physics in studies of atoms and of nuclear energy investigations in attempts to understand nucleons. While useful, such investigations may not be sufficiently directly aimed at the most truth revealing facts and may have to rely on help from meson physics which has already been partially successful in forming views on nuclear forces.

CONCLUDING REMARKS

It has been attempted above to give an account of the principal lines of investigation on nuclear forces. An answer to the main question has not been arrived at, but it can be said that one knows the parameters which describe consequences of nuclear interactions in the low energy field, that with a high degree of probability nuclear forces have their origin in something having to do with mesons, perhaps in some meson theory of a reasonably standard type which will form a logical completion of the present $PS(ps)$ attempts, that for the high energy phenomena one can hardly get along without considering the composite nature of a nucleon. The fact that the range parameters of nuclear forces have the order of magnitude e^2/mc^2 may have its partial explanation in the mass of the π-meson, but it may be also that some more fundamental approach such as that of Heisenberg[167] comes under consideration and that it will not suffice to deal with the problem by means of a special field designed for the purpose of explaining nuclear forces rather than by means of the interplay of a number of fields. The fact that it has proved possible to obtain results agreeing with experiment in electrodynamics is hardly proof that one can circumvent the danger of infinities in the field of nuclear forces. In the development of the theory of electrodynamics there has been the strong support of complete knowledge of the behavior of electromagnetic fields in the domain of classical theories as well as the invaluable aid of such developments as Bohr's correspondence principle, elaborate tests on the validity of ordinary quantum-mechanical formulas for emission and absorption in the nonrelativistic region, the Klein-Nishina formula, the Bethe-Heitler formula, etc. In the field of nuclear forces the approach is more abstract. No direct measure of a field quantity has been made in the same sense in which one measures an electric or magnetic field, and the body of factual knowledge available for theory is smaller.

[166] R. Serber, Phys. Rev. **72**, 1008 (1947); Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

[167] W. Heisenberg, Festschrift zur Feier des 200 jährigen Bestehens der Akademie der Wiss. in Göettingen, 1951; Comm. on Pure and Appl. Math. **4**, No. 1 (1951); Z. Naturforsch. **5a**, 367 (1950); **5a**, 251 (1950); **6a**, 281 (1951); M. Fierz, Helv. Phys. Acta **23**, 731 (1950); E. Stueckelberg and D. Rivier, Helv. Phys. Acta **23**, 215 (1950).

The New Physics Building at the Ohio State University

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(Received July 23, 1952)

The new Physics Building at the Ohio State University is described and pictured. Floor plans are shown and special features of the building are discussed.

IN 1945 the Ohio Legislature made an appropriation for the construction of a new physics building at The Ohio State University, but the steel shortage and the immediate priority assigned for this building suggested that it would be at least two years before actual construction could begin. One year later the chairman of the Department of Physics and Astronomy appointed a committee to make a thorough survey of the space needs of the entire department. This committee obtained from the members of the permanent staff estimates of the space demands of every part of the department's program, both as it then existed and as it was projected on the basis of the growth that could be expected within a "few" years. The committee studied available reports[1] on other physics buildings, discussed sizes and types of lecture rooms, and suggested an approximate location which each of the various types of rooms should have in a new building. These specifications included such special requirements as the need for areas in which there were no windows. Finally the committee reported its findings to the university architect, whose office prepared sketches for a building that would provide the space requirements of some 200 000 square feet as outlined by the building committee and would, in general, satisfy all other needs.

The housing requirements can be understood by consideration of the present size of the department. In the school year 1951–1952, the active permanent staff totaled 22, exclusive of those associated with astronomy and optometry groups. There were 100 undergraduate majors in physics, half of whom were enrolled in the College of Arts and Science and half in the College of Engineering. There were somewhat more than 150 graduate students. Of the latter, 30 assisted in the teaching activities and 35 others were employed on a part-time basis to participate in sponsored research investigations. About 20 technical assistants had full time employment in shops, laboratories, and other service facilities.

The appropriation for the physics building, materially increased by the Ohio Legislature in 1947, was sufficient to build only approximately one-third of the projected building when materials became available. The building committee was charged with the responsibility of recommending which of the activities of the department should be housed in the new building and which would have to remain in Mendenhall Laboratory of Physics. The site chosen was close to the chemistry and engineering buildings of the campus. This site causes some inconvenience because of its separation from Mendenhall Laboratory, but its choice is part of a long-range plan of bringing the physical science departments together in the vicinity of the so-called "engineering quadrangle." After considerable study the staff decided that space should be provided in the new building for two lecture rooms, three additional classrooms, offices for most of the physics staff, advanced laboratories, and most of the research programs. To remain in Mendenhall Laboratory were the instrument shop, the elementary laboratories, the library, several classrooms, the lecture room and other facilities of a descriptive course in physics and astronomy, and certain research laboratories for which special provisions had previously been made.

[1] The following references describe physics buildings or facilities at various institutions and are of interest to building committees. B. H. Dickinson and R. H. Noble, Am. J. Phys. **18**, 378 (1950); C. E. Howe and F. G. Tucker, Am. J. Phys. **17**, 245 (1949); Design of Instructional Laboratories, A Symposium, Am. J. Phys. **13**, 182 (1945), B. J. Spence, Am. J. Phys. **11**, 208 (1943); M. Masius, Am. J. Phys. **10**, 307 (1942); W. P. Davey, Am. J. Phys. **10**, 102 (1942). This last reference contains a complete bibliography of previous building reports.

FIG. 1. The Ohio State University Physics Building viewed looking about 30° north of west.

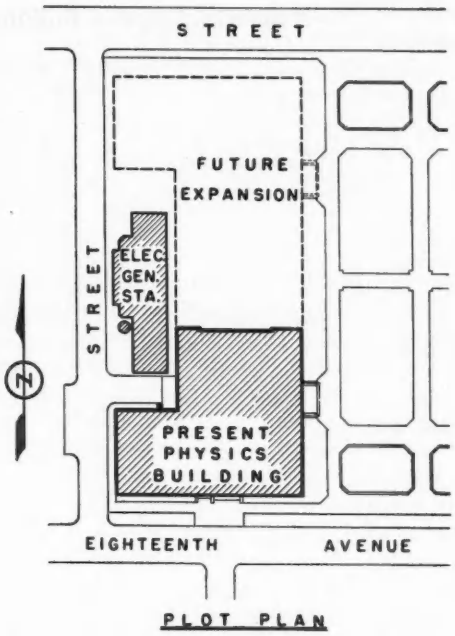

FIG. 2. Plot plan showing the relation between the existing portion of the physics building and its proposed extension.

GENERAL FEATURES

During the summer of 1948 construction was begun on a four-story-and-basement structure for the physics department; this building was finally occupied early in 1951. The new Physics Building is of modified classical Renaissance architecture, as are other buildings in the area. The exterior is red brick, trimmed in limestone and with green brick spandrels below the windows on the second and third floors. Aluminum casement windows are used throughout the structure and Venetian blinds have been provided for each window, except for those in certain laboratories equipped with light-tight shades. Figure 1 shows the front of the building as viewed by a camera pointed approximately 30 degrees north of west. The building is 121 ft long, 92 ft wide, and has a wing 42 ft by 66 ft. Future plans are to complete the building by extending it northward as is indicated in Fig. 2.

One of the objectives sought in the design of the building was flexibility. Toward this end there are generous vertical service shafts which carry electrical conductors, water, gas, compressed air, and connections to a vacuum pump to the various laboratories. The design of these service shafts is such that it is relatively easy to add any of the special facilities which a physicist is likely to require in his research laboratory. The internal walls of the building are of unglazed yellow tile. These walls, while not portable, do not bear any load and are fairly economically moved.

At the present time the roof of the building is flat and has associated with it certain facilities for making meteorological measurements. A platform on the shed which houses the elevator controls is used for making observations and for balloon launchings. It is planned that a fifth floor and a sloping copper roof will be added so that this building will harmonize with other buildings in the engineering quadrangle. Figure 3, which is a sectional drawing of the building, includes the existing portion and these contemplated additions.

In planning the building it was decided that an effort would be made to put the facilities used by large numbers of students on the first and second floors so far as possible. Not only is this more convenient for students, but it provides quieter research rooms and minimizes confusion in the halls. Accordingly, the lecture rooms and the largest of the three recitation rooms are located on the first floor, and the other classrooms on the second floor. Most of the advanced laboratories are on the second and third floors. The basement and the fourth floors are largely

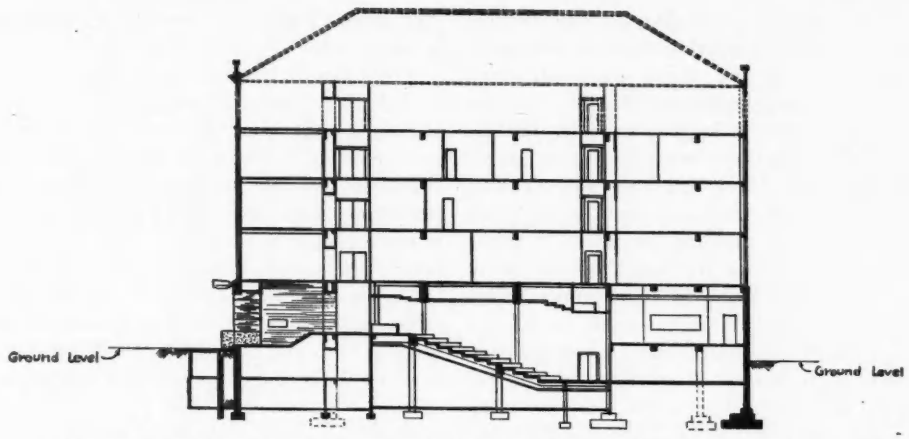

FIG. 3. North-South section of the physics building. In addition to the existing four floors and basement, a proposed fifth floor and sloping roof are indicated.

devoted to research activities, although one lecture room does reduce the basement area available for this purpose; classes in meteorology are taught on the fourth floor.

LECTURE ROOMS

Inasmuch as the two lecture rooms and their common apparatus-storage rooms require a large volume of the space and because these lecture rooms must be conveniently accessible to the public, their locations were the first selected and then the rest of the building was planned around them. The lecture rooms have 235 and 116 seats, respectively.

Large Lecture Room.—Figure 4 is a photograph of the interior of the larger lecture room, taken from a back corner. It is located so that the students ordinarily enter directly from the first floor level at the rear of the lecture room. The slope is such that it drops 11 ft in 50 ft, with the result that the lecturer stands at the basement floor level. This is illustrated in Fig. 3. This slope is sufficient to permit occupants of all seats to have a reasonably good view of the lecture table and of apparatus placed on the floor near the table. This relatively steep slope has the disadvantage of making it difficult to conduct examinations since students may view the papers of occupants of the row ahead of them. There is an entrance at the lecture table level from the basement hall on one side and an entrance from the lecture preparation room on the other. The ceiling is acoustically treated. Seats are of the fixed-arm single pedestal type, fastened to the floor. At the rear of the room is a projection table equipped for either motion pictures or slides. Ventilation is provided by an air conditioning system.

The lecture table consists of a fixed section 14 ft long with a top 39.5 in. wide. In this fixed section there are a sink, hot and cold running water, several outlets for gas, compressed air, and vacuum, a steam line, and several drawers and cupboards. Also in the fixed section is an electric distribution panel which permits the use

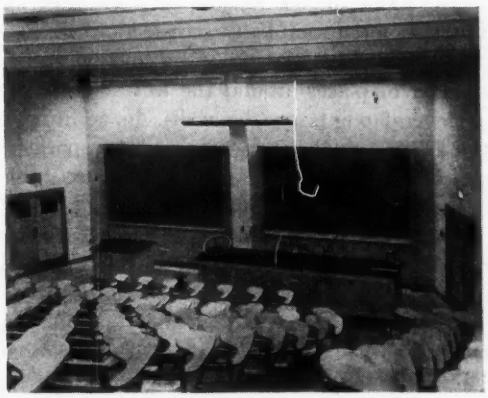

FIG. 4. View of the large lecture room, taken from the south-east corner.

of any dc voltage available at the central switchboard. Built into this distribution panel is a Variac useful for many demonstrations involving alternating voltages. The fixed section of the lecture table is located considerably off center. As many as three movable sections, each four feet long, may be rolled into place next to the fixed section. A jumper system with Jones plugs permits the supplying of power to these movable sections from the fixed section in a matter of seconds. The movable sections may be prepared in the lecture preparation room and wheeled into the designated space when desired. In some cases it is most convenient to use the space normally occupied by one or more movable sections for the location of other apparatus. For example, in demonstrating the conservation of angular momentum with a turntable it is convenient to place the rotating platform in the position commonly occupied by one of the movable sections. In this way the students have a good view of the entire apparatus and the lecturer has room to maneuver.

Behind the lecture table there are two 12-ft green chalk boards, each with a large fixed section and a counter-balanced movable section which slides vertically in front of the fixed section. Directly above the lecture table there is a catwalk to which there is access from the lecture preparation room. This is indicated in Fig. 3. From this catwalk it is possible to suspend such equipment as pendulums, an Atwood machine, and other apparatus.

The room has no outside windows. Illumination is by fluorescent lights recessed in the ceiling. These lights can be turned on and off from a number of places around the lecture table and at several points on the walls. In addition to the fluorescent lights there are three spotlights which are directed toward the lecture table; each is operated by a separate switch.

Small Lecture Room.—The smaller lecture room also has a sloping floor but the pitch is considerably less than that of the larger room; actually the lecturer stands only about four feet below the floor level at the rear of the room. This room also has no outside windows; the fluorescent lights recessed in the acoustic ceiling may be controlled from a number of points in the room, including several locations on the lecture table.

The room has a fixed lecture table 12 ft long and 39.5 in. wide. This fixed section contains the same facilities as does the one in the larger lecture room. It also is off center in the room so that one movable section, indentical to those previously described, can be wheeled in from the lecture preparation room. The slope of the floor makes the lecturer's position two feet below the first floor elevation and the movable table must, therefore, be rolled down a ramp to bring it into the room. The small lecture room has one large fixed green chalk board, and a counter-balanced movable board which slides vertically in front of the fixed section.

Apparatus Storage.—A common lecture-preparation room serves both lecture rooms. The lecture-preparation area is partly at the basement level and partly at the first floor level. An inside stair connects the two levels and there is a dumb-waiter for the transportation of small equipment. For very heavy equipment it is necessary to wheel the apparatus a few feet to the elevator.

FLOOR PLANS

First Floor.—The plan of the first floor is shown in Fig. 5. In addition to the lecture rooms, apparatus storage and one classroom, there are also located the department office, the chairman's office, one other office, a large staff room, and a suite of rooms devoted to low temperature research.

The department office, a room approximately 21 ft square, contains the desks of the departmental secretaries, the inevitable filing cabinets, and the department mail box. Connected to the office on one side and accessible only through the office is a small room of some 210-ft^2 area in which mimeograph equipment and departmental supplies are kept, while on the other side is the office of the chairman, which may be entered either from the departmental office or from the hall. Telephone wires to various offices and laboratories in the building are run in parallel to an answering box in the main office, so that when any telephone in the building is not answered promptly a secretary can take a message.

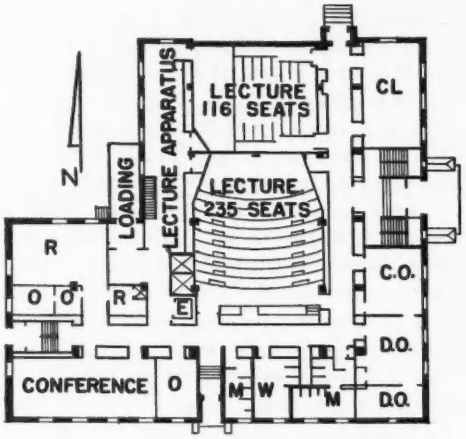

FIG. 5. First floor plan.

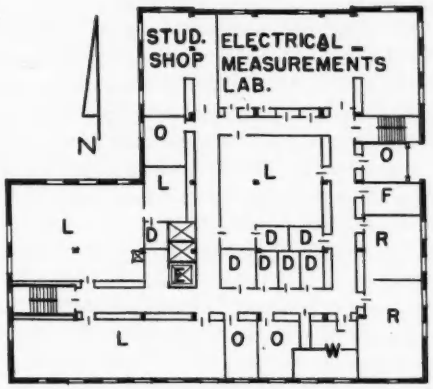

FIG. 8. Third floor plan.

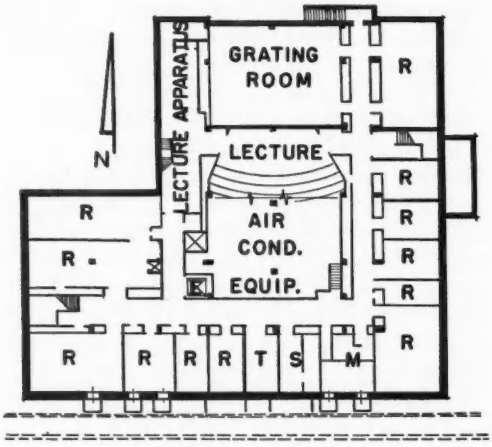

FIG. 6. Basement plan.

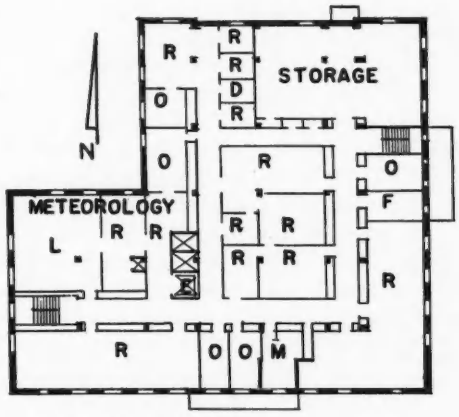

FIG. 9. Fourth floor plan.

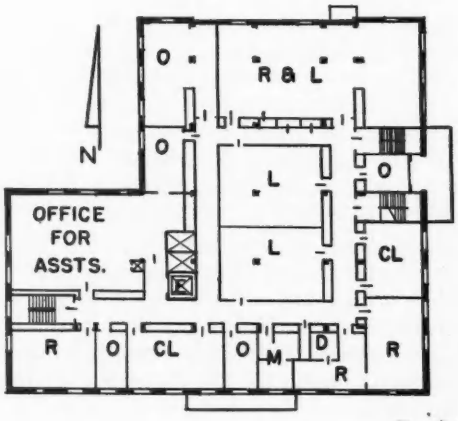

FIG. 7. Second floor plan.

Key to designations on floor plans in Figs. 5 through 9

Cl Classroom
C.O. Chairman's office
D.O. Departmental office
D Darkroom
E Elevator
F Fan room
L Laboratory for experimental courses
M Men's room
O Office
R Research room
S Main switchboard and generators
T Transformer
W Women's room

Basement.—The arrangement of the basement rooms is shown in Fig. 6. The large lecture room and the apparatus storage room occupy roughly half of the available area. Most of the remainder of the space is used for various researches. The largest of these rooms has been kept free from posts so that it may be used for a ten-meter Rowland circle. In all of the basement research rooms the floors are independent of the building itself. They are supported on separate footings and are mechanically insulated from the building by cork pads. Most of the basement rooms have no windows; a few have areaway lighting. Also in the basement is the main switchboard and generator room.

The area under the sloping floor of the large lecture room has been utilized for the location of much of the mechanical equipment required for heating and ventilating the building and for air conditioning the large lecture room. As is indicated in Fig. 3, there is a sub-basement floor which is about 6 feet below the ordinary basement floor level. A false ceiling made of sound absorbing material on the underside of the floor of the large lecture room prevents the noise of the mechanical equipment from disturbing occupants of the lecture room.

Upper Floors.—The plans of the second, third, and fourth floors are shown in Figs. 7, 8, and 9, respectively. In general, the use to which each room is put is indicated on the floor plan.

Each of the offices is approximately 20 ft long and 10 ft wide. The length of these rooms is determined by the corridor-to-outside-wall distance; it is somewhat greater than is actually required for a satisfactory office. It is proposed that when the building is enlarged, a wing of somewhat smaller offices be included. Each office is equipped with a green chalk board, desks, a table, chairs, bookshelves, a coat tree, and filing cabinets. The two offices directly above the main entrance to the building have large areas of glass brick in the outside east wall.

ELECTRICAL FACILITIES

Because experimental physics requires the use of large amounts of electrical power, much attention was given to the facilities needed for distribution of electricity.

All rooms are supplied with several 110-v alternating current outlets, protected through circuit breakers in the halls near the room. Three-phase 208-v power is provided in certain rooms where its use was foreseen. It is available through the main switchboard to all laboratories.

Direct current is provided by a dual generator from which 125- or 250-v potentials may be obtained. Although it was decided that storage batteries should not be included in the original installation of electrical equipment, it was anticipated that the time would come when they would be required, and therefore distribution facilities for battery power were included in the switchboard purchased.

Generator dc, battery dc (in the future), three-phase ac, and supplementary single-phase ac power are distributed through the main switchboard in the basement to eleven large distribution panels. There is at least one of these on each floor. Special distribution panels are provided in each of the lecture rooms and in the electrical measurements laboratory. Through the distribution panels power may be supplied to small subdistribution panels located in the various laboratories. The number in each laboratory varies from one in the small research rooms to 26 in the electrical measurements laboratory. The electrical distribution system and the associated equipment were purchased from the Standard Electric Time Company.

SERVICE FACILITIES

In addition to electrical facilities, most of the laboratory rooms are supplied with a small sink, hot and cold water, gas, compressed air, and a vacuum line.

All interior rooms of the building are heated and ventilated by a forced-air system. Provision is made for air conditioning these rooms, but at present refrigeration is provided only for the large lecture room. External rooms are heated by hot water radiators and forced air.

The elevator, which is key operated, is slightly over 6 ft square. The elevator door has a sliding section for ordinary use, but the entire door can be swung open on hinges in such a way as to admit an object a full six ft wide, thus making the elevator useful for the movement of large equipment.

At the rear of the first floor there is a receiving room adjacent to a loading platform, to which trucks can back to discharge their loads. This loading platform is conveniently close to the elevator and the lecture-preparation rooms.

RECOGNITIONS AND ACKNOWLEDGMENTS

It is not feasible to acknowledge here the work of every person who made substantial contributions in connection with the new building. However, special recognition must be given to a few. The vital function of preparing and presenting to the Legislature of the State of Ohio the case for a new physics building was performed by President Howard L. Bevis, Vice President and Business Manager Jacob B. Taylor, and Dean Emeritus Alpheus W. Smith, at that time chairman of the Department of Physics and Astronomy. Professor H. H. Nielsen, who succeeded Dean Smith as department chairman, was active in many phases of the building program. University Architect Howard Dwight Smith and his staff designed the building and supervised its construction. Many of the department's interests were in the hands of the building committee, whose members were as follows: the late Professor Alva W. Smith, who was chairman until 1948, M. L. Pool, G. Shortley, J. A. Hynek, D. Williams, and the authors. Many of our other colleagues made valuable suggestions and contributions.

The authors are indebted to H. F. Reichard, Associate University Architect, for his aid in the preparation of the drawings used in this paper.

The Tape Recordings of Important Speeches

BY

E. U. CONDON, K. K. DARROW, E. FERMI, J. C. SLATER

Free tape recordings are available on loan to members of the Association who wish to use them for presentation to classes, seminars, or science clubs. The subjects are four of the six lectures presented as a Symposium on Physics Today at the Twentieth Anniversary meeting of the American Institute of Physics, Chicago, Illinois, on October 25, 1951, as follows:

The atom. E. U. CONDON.
Physics as science and art. K. K. DARROW.
The nucleus. E. FERMI.
The solid state. J. C. SLATER.

Recordings are now available in two forms:

1. *Double track.* Each speech is complete on one reel, which must be played at 3¾ inches per sec. No rewinding is necessary. Track No. 1 is played to the end, then the reels are reversed (and turned over) to play track No. 2. This procedure rewinds the tape into its original condition.
2. *Single track.* Two 7-inch reels are required for each address. They must be played at 7½ inches per sec, and, of course, rewound to restore them to their original condition.

Requests for recordings should be made to THOMAS H. OSGOOD, Editor, *American Journal of Physics*, Michigan State College, East Lansing, Michigan. To allow some flexibility in scheduling, a request should specify a preferred date and two alternative acceptable dates, as well as the type of recording (single or double track, or either).

No charge is made for the loan of recordings. The borrower is expected, however, to pay the return postage to the Editor's office or to any other address the Editor specifies. Transportation is to be by first class mail, special delivery, which will cost less than one dollar per reel at present rates. Reels will be mailed in convenient reusable boxes.

Prospective borrowers may consult the published papers in *Physics Today*, November, 1951, January, 1952, and March, 1952.

Three Lecture Room Demonstrations in the Field of Electricity

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(Received July 2, 1952)

Semiquantitative verification of the impedance formula for series circuits containing inductance and capacitance, phase relations in similar circuits including the case of resonance, and a triode oscillator operating in the frequency range from 2 cycles/sec to 20 000 cycles/sec, all based on a dissectible transformer, are described.

THERE is on the market a humble instrument made in the USA by Cenco[1] and in Germany by Phywe,[2] a dissectible transformer which, with a few accessories and interchangable coils, can be of great aid in teaching electricity. Besides those experiments described in the "Directions"[3] by Cenco there are quite a number of different experiments, very simple ones, which are based on this transformer. The above quoted "Directions" lists six different groups of experiments which may be performed with the transformer. However, many others, not listed, are possible. Just to mention a few: Forces on Parallel Conductors, Principle of AC Meters, Inductive and Capacitive Reactance, Phase Relations, etc. The demonstrations are not only qualitative but in many instances semiquantitative too.

In the following, three experiments are described based on the transformer.

The parts of the transformer are: U-shaped laminated iron core; laminated iron armature; mounting clamp, consisting of top and base and clamping bolts and nuts to hold the transformer together; three each of the following coils: 6, 300, 600, 1200, 12 000, and 18 000 turns. These coils fit the need better than those supplied by Cenco. However, on special request, Cenco is ready to furnish any purchaser with the above coils.

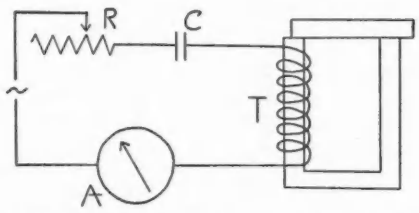

FIG. 1. Schematic diagram of Experiment No. 1. R = rheostat 360 ohms/1 amp. $C = 10\ \mu$f condenser. $T = 1200$-turn coil. A = ac ammeter, range up to 1 amp.

EXPERIMENT NO. 1. SEMIQUANTITATIVE VERIFICATION OF THE IMPEDANCE FORMULA

Whenever a series circuit contains both inductance and capacitance an important special case arises whenever L, C, and f have such values that $2\pi fL = 1/2\pi fC$, that is when $X_L = X_C$, then $X = 0$, $Z = R$, and $\varphi = \tan^{-1} X/R = 0$. The impedance of the circuit is equal simply to its resistance, and the current is in phase with the potential difference across the circuit. These circumstances can result in unduly high currents and potential differences in parts of the circuit. The schematic diagram and the necessary data of the circuit for this experiment are shown on Fig. 1.

In performing the experiment, the rheostat, the 10-μf condenser, and the ammeter are connected in series to the 220-v ac power source. The rheostat should be adjusted to allow the flow of a current of 0.4 amp through the circuit.

For the next step the condenser should be replaced by a 1200-turn coil on the iron core. The ammeter will indicate a current of about 0.5 amp. If the armature is gradually pushed over the U core the current drops step-by-step to 0.2 amp. If now the rheostat, condenser, and coil are connected in series, the current is even higher than in the previous case. By changing the position of the armature, there is found a certain position at which the ammeter indicates maximum current. Moving the arma-

* Now at Thiel College, Greenville, Pennsylvania.
[1] Cenco, Scientific Instruments and Laboratory Supplies, Catalog J—150, p. 1243.
[2] Roller-Prix, *Wechselströme* (Phywe edition, 1938).
[3] Cenco, Directions for Use of Cat. No. 80280, Dissectible Transformer.

ture from this position in either direction causes the current to drop. Since $Z=\sqrt{\{R^2+(X_L-X_C)^2\}}$ and for the case of resonance $X_L=X_C$, it is possible to compute the inductance of the coil for this particular position of the armature and to point out that when the armature is being moved in either of the directions the drop of the current indicated by the meter is very similar. On this basis the semiquantitative diagram current *vs* inductance can be drawn (Fig. 2).

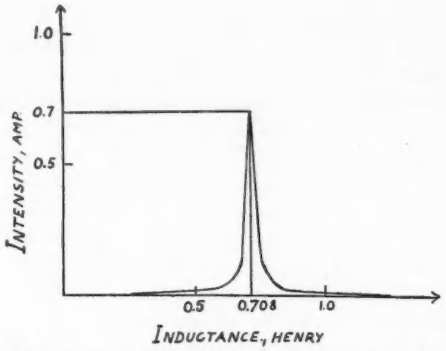

FIG. 2. Resonance curve. Current in a series circuit as a function of inductance illustrating series resonance at the value of $L=0.708$ henry that makes $X_L=X_C$, if $C=10$ μf.

EXPERIMENT NO. 2. PHASE RELATIONS IN SERIES CIRCUITS

This experiment is to demonstrate the phase of the current in relation to that of the potential difference. If $v=V_m \sin 2\pi ft$ and $i=I_m \sin(2\pi ft-\varphi)$, then

$$\varphi=\tan^{-1}X/R.$$

Since the reactance X may be either positive or negative, the same is true for the phase angle φ. If $X_L>X_C$, φ is positive, the current is said to lag the potential difference. On the other hand, if $X_L<X_C$, φ is negative, the current leads the potential difference. For the case of resonance $X_L=X_C$ and $\varphi=0$, current and potential difference are in phase.

The phase relations in this experiment are shown by a phase indicator (Fig. 3) which is placed in front of the coils T_A and T_B. The phase indicator is *not* connected to either of the circuits. The schematic diagram and the necessary data of the circuits are shown on Fig. 4.

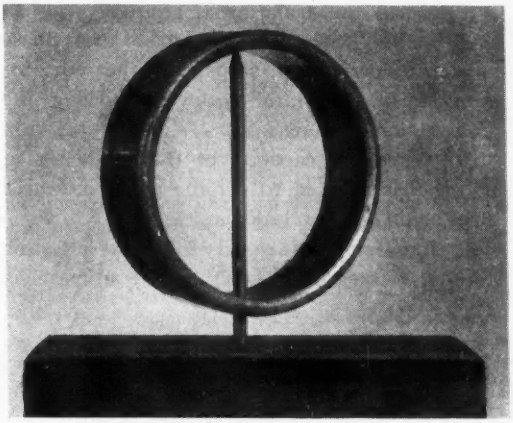

FIG. 3. Phase indicator. Diameter of aluminum ring 7 cm. The ring is pivoted on a nail. The nail and hole are coated by lacquer to eliminate shorting along the axis of rotation. A coil of a few turns, shorted, pivoted like the ring, will do equally well.

The author performed the demonstration through the following steps:

(a) Circuit A contains an ohmic resistance (300 ohms) and a coil with iron core. The ohmic resistance is predominant, and the current is well in phase with the potential difference. In circuit B the rheostat is adjusted to approximately 200 ohms and the 1200-turn coil has a U-shaped core. This circuit is predominantly nonohmic. The phase indicator rotates clockwise indicating that the current in B lags behind the potential difference and $\varphi_B>\varphi_A$.

(b) When the condenser of 8μf is added to circuit B the phase indicator turns counterclockwise indicating that $\varphi_B<\varphi_A$.

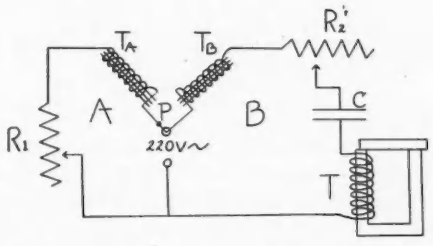

FIG. 4. Schematic diagram of Experiment No. 2. R_1=rheostat, 300 ohms/1 amp. R_2=rheostat, 300 ohms/1 amp. T_A=600-turn coil, iron core. T_B=600-turn coil, iron core. T=1200-turn coil, U core and armature. C=set of 2 condensers of 4 μf and 1 condenser of 2 μf, connected parallel. P=phase indicator.

(c) When the armature is gradually pushed over the U core, the rotation of the phase indicator slows down, stops, and the indicator even starts to rotate clockwise showing the relative change of φ_B.

(d) When the capacity of the condenser in circuit B is reduced to $4\mu f$ the phase indicator turns again counterclockwise.

(e) Increasing the capacity to $6\mu f$ the indicator stops or rotates very slowly. Slight adjustment of the armature over the U core brings the indicator to full stop, showing that $\varphi_B = \varphi_A \cong 0$ and in circuit B the condition $2\pi fL = 1/2\pi fC$ is fulfilled.

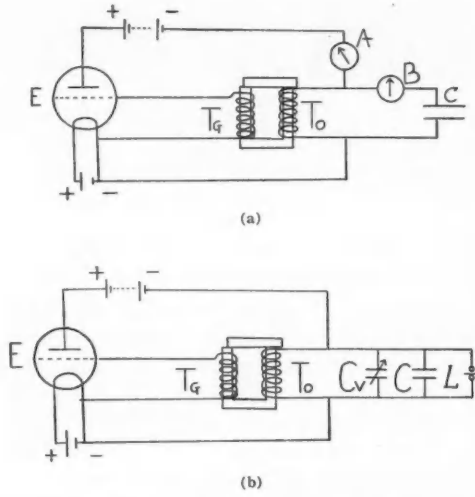

FIG. 5. Schematic diagrams of Experiment No. 3. (a) The diagram for the first part of the experiment. A = milliammeter. B = galvanometer, zero-center instrument. C = set of 2 condensers of 4 μf and of 2 condensers of 2 μf, connected parallel. E = No. 801 or any other triode. T_G = 12 000-turn coil. T_0 = 12 000-turn coil. Both coils on U core with armature.

(b) The diagram for the second part of the experiment. C_V = variable condenser, 500 mμf. C = set of condensers of 0.5 μf, 2 μf, and 4 μf. (All connected parallel.) E = triode, L = loud-speaker. T_G = coil 12 000/1200 turns, respectively. T_0 = coil, 1200/600 turns, respectively.

EXPERIMENT No. 3. TRIODE OSCILLATOR FOR FREQUENCIES OF 2–20 000 CYCLES/SEC

For an average student it has been always difficult to realize what is going on in an oscillating circuit. The experiment described in the following brings the facts very close to the student's mind. The student will be able to "visualize" the oscillations for frequencies between 2−15 cycles per sec, and in the second part of the experiment even to "hear" them for frequencies between 20−20 000 cycles per sec. The schematic diagram and the necessary data of the circuits are shown in Fig. 5.

This experiment has two parts: The oscillations in the 2−15 cycles region are shown by the pulsations of the pointer of a galvanometer. The inertia of the galvanometer prevented visible movement at higher frequencies, therefore in the second part of the experiment, the galvanometer is replaced by a loud-speaker and the sound indicates the frequencies of the oscillations.

In the first part of the experiment (Fig. 5a), the milliammeter in the plate circuit shows pulsating dc and the galvanometer in the oscillating circuit (zero-center instrument) an ac of undamped amplitudes. If the inductance of the coil is changed by pulling off gradually the armature of the U core, or by reducing the capactance by 2−10μf, or by doing both, the frequency increases. If the inertia of the galvanometer is not too high it can follow vibrations up to 15 cycles per sec.

In the second part of this demonstration the coil in the oscillator circuit is replaced by one with 1200 turns, the zero-center instrument by a loud-speaker, and a 500-mμf variable condenser is connected parallel to the condensers. The frequencies are in the audible range. The loud-speaker gives a sound similar to that of an idling gasoline engine. Changing inductance and/or capacitance causes the frequency to increase. If the coil in the grid circuit is replaced by one of 1200 turns and in the oscillator circuit by one of 600 turns, the number of oscillations can trespass the range of audible frequencies.

NOTES AND DISCUSSION

Note in Memory of Blind John Gough*

ALLEN L. KING
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ONE-HUNDRED-AND-FIFTY years ago on November 16, 1802 John Gough, the blind philosopher of Kendal (England), wrote to Dr. Holme, a Vice President and later President of the Manchester Literary and Philosophical Society:

> The substance called Caoutchouc, or Indian Rubber, possesses a singular property; which, I believe, has never been taken notice of in print, at least by any English writer; the present letter contains my experiments and reflections on the subject; and should they appear to deserve the attention of your philosophical friends, I am certain you will take the trouble of communicating the paper to the Literary and Philosophical Society of Manchester.
>
> The property I am about to describe, depends on the temperature of the Caoutchouc. . . .

Thus begins this most interesting letter[1] by an unusual man. John Gough was born on January 17, 1757 at Kendal, England. At the age of three he lost his sight from an attack of smallpox and subsequently he trained his senses of touch and hearing to an unusual degree. George Bewly, cousin to John Dalton and master of the Friends' school in Kendal, influenced John Gough to study natural history. At the age of eight he began studying plants by touch and in a few years he became very expert in identifying them. He handled the plants rapidly from foot or stalk upward, examined the stamens and pistils with the tip of his tongue, and detected the finest hairs with his lower lip. His lower lip! We can now appreciate his gentle admonition regarding any criticism of the manner of performing his experiments:

> Perhaps it will be said, that the preceding experiment is conducted in a negligent manner; that a person, who wishes accuracy, will not trust his own sense of feeling in inquiries of this description, but will contrive to employ a thermometer in the business. Should the objection be started, the answer to it is obvious; for the experiment in its present state demonstrates the reality of a singular fact; by convincing that sense, which is the only direct judge in the case, that the temperature of a piece of Caoutchouc may be changed, by compelling it to change its dimensions. The use of a Thermometer determines the relative magnitudes of these variations, by referring the question of temperature to the eye; experiments of this sort are therefore of a mathematical nature, and afford a kind of knowledge with which we have nothing to do at present; for we are not inquiring after proportions, but endeavoring to establish the certainty of a fact, which may assist in discovering the reason of the uncommon elasticity observable in Caoutchouc.

Although Gough's brief reference to mathematics in this quotation suggests but little acquaintance with the field, actually he was sufficiently expert to teach a number of private students the elements of mathematics including the fluxions. Among his students was John Dalton whom he later recommended for a professorship of mathematics and natural philosophy at New College in Manchester. Dalton's first public statement on chemical proportions immediately precedes Gough's letter in the first volume of the second series of the *Memoirs of the Manchester Literary and Philosophical Society* and, furthermore, his important *Experimental Enquiry into the Proportion of the Several Gases or Elastics Fluids Constituting the Atmosphere* was read November 12, 1802!

This note, however, is written in honor of John Gough, the blind philosopher and teacher, who described the peculiar thermoelastic properties of rubber for the first time. His experiments are three in number:

> Experiment 1. Hold one end of the slip [of prepared Caoutchouc] between the thumb and forefinger of each hand; bring the middle of the piece into slight contact with the edges of the lips; taking care to keep it straight at the time, but not to stretch it much beyond its natural length: after taking these preparatory steps, extend the slip suddenly; and you immediately perceive a sensation of warmth in that part of the mouth which touches it, arising from an augmentation of temperature in the Caoutchouc: for this resin evidently grows warmer the further it is extended; and the edges of the lips possess a high degree of sensibility, which enables them to discover these changes with greater facility than other parts of the body. The increase of temperature, which is perceived upon extending a piece of Caoutchouc, may be destroyed in an instant, by permitting the slip to contract again. . . .

Indeed if a stretched rubber band is allowed to come to thermal equilibrium with room air, it will acquire a lower temperature than that of the room on contracting suddenly. This thermoelastic effect was difficult to explain until the role of entropy in the process was fully realized. On stretching the rubber band the long chains of macromolecules in it become partially aligned and, therefore, more ordered; thus the band gives up energy by heat flow and its temperature rises. The band feels warm. When the band contracts the reverse process takes place; the chains become less ordered and thus require an inflow of thermal energy to bring the band to equilibrium again. Heat flows from the lips, or other sensing element, into the rubber band and the band feels cold.

> Experiment 2. If one end of a slip of Caoutchouc be fastened to a rod of metal or wood, and a weight be fixed to the other extremity, in order to keep it in a vertical position; the thong will be found to become shorter with heat and longer with cold. . . .

This description is followed by an explanation for the foregoing effects in terms of a caloric fluid. It must be remembered, however, that Rumford had described his crucial observations only three years earlier and Joule had not been born.

> Experiment 3. If a thong of Caoutchouc be stretched, in water warmer than itself, it retains its elasticity unimpaired; on the contrary, if the experiment be made in water colder than itself, it loses part of its retractile power, being unable to recover its former figure; but let the thong be placed in hot water, while it remains extended for want of spring, and the heat will immediately make it contract briskly.

Apparently these curious effects were not nearly as strange as they seemed fifty years later; for they could be readily understood in terms of the then popular caloric theory. In Gough's words:

> The object of the present letter is to demonstrate, that the faculty of this body (Caoutchouc) to absorb the calorific principle, may be lessened, by forcibly diminishing the magnitudes of its pores; . . .

It was James Prescott Joule, prime destroyer of the caloric theory, who resurrected Gough's work, repeated his experiments in a quantitative manner, and set the stage for the future developments in the theory of rubber-like elasticity.[2]

Joule was born on Christmas eve in 1818, six-and-one-half years before Gough's death and because of delicate health received his early education at home. In 1835 he went to Manchester to study under Dalton. Dalton taught him the elements of mathematics, and thus in some measure Gough may be said to have had an early influence on Joule through Dalton. From Dalton, Joule received his first inducement to become a scientific investigator. In 1859 he described his experiments on rubber and thus established his relationship to the curious thermoelastic effects which were originally observed by John Gough.

* *Dictionary of National Biography*, v. XIII, p. 428, v. XXII, p. 277; v. XXX, p. 208 (Macmillan and Company, New York, 1888-1892).

[1] John Gough, "A description of a property of caoutchouc," *Memoirs of the Literary and Philosophical Society of Manchester*, 2nd series, vol. 1, p. 288 (1805).

[2] J. P. Joule, "On some thermodynamic properties of solids," *Joule's Scientific Papers*, vol. 1 (The Physical Society of London, 1884), p. 429.

A Set-Up for the Vibrating Wire Experiment

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College of Agriculture and Mechanic Arts, Mayagues, Puerto Rico

A WIRE is clamped at one end and the tension is made variable by hanging weights from the free end after passing over a movable and a fixed frictionless pulley, as in Fig. 1. The ends of the wire are connected as load to

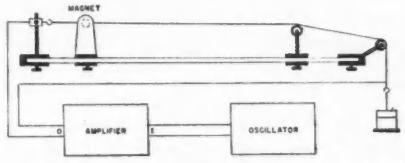

FIG. 1. Wiring diagram of magnetically driven vibrating wire.

the output of an audio amplifier whose input is fed by an audio oscillator. A horizontal magnetic field is applied to the wire by setting a magnetron magnet astraddle it. Since the wire is connected to the output of the amplifier, an alternating current flows through it having the same frequency as the signal from the audio oscillator. Since the magnetic field is perpendicular to a small portion of the wire it is subjected to a vertical periodic force.

The amplitude of vibration is controlled by the setting of the gain control in the amplifier and by varying the distance from the magnet to the fixed end of the wire.

The audio amplifier has two 6L6 power tubes in push-pull, and delivers a rated power of 25 w on the proper load. This power is enough to operate the apparatus nicely. Since the amplifier has a universal output transformer it was possible to match properly its electrical output impedance to that offered by the wire.

The sinusoidal signal applied to the input of the amplifier was supplied by a Hewlett-Packard low frequency oscillator.

The strength of the magnet is critical and since the magnetron magnet has a variable gap it was adjusted for best performance.

Many types of wires were tried and the best results were obtained by using the wire from a Bates stapler.

This set-up was a success in teaching the laws of a vibrating wire to our class in college physics. I want to express sincere thanks and appreciation to Professor Roberto Ortiz for his advice and to Professor Fabio Gonzalez for his help in testing this piece of equipment.

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 3 MARCH, 1953

LETTERS TO THE EDITOR

Sinusoidal Representation of Harmonic Motion —An Introductory Demonstration

THE beginner is often hard-put to see at once the relation between the fictitious *reference circle*, which is a purely artificial construction (for when a body describes harmonic motion there is usually no associated body which actually travels with circular motion), the *actual harmonic motion*, and the "*wave*" *motion* which leads to the sine curve construction. Reference to a diagram alone, I have found, leaves a good many students a little puzzled. The connection can be readily realized by a very simple artifice.

Support a coil spring with load on a not-too-heavy standard. Having drawn a reference circle on the blackboard, extend the horizontal diameter to represent the time axis. Hold the loaded spring against the blackboard diagram with the mass at the center of the circle (equilibrium position). Impart a motion of amplitude equal to the circle radius. Now walk uniformly parallel to the time axis. The path of the vibration is very elegantly spread out to form the curve which heretofore appeared to be somewhat artificially created. Indeed, a very *natural* connection appears now to exist between the particle motion and the wave. With a little care the curve can be actually traced upon the board by fixing a piece of chalk to the vibrating mass, with due caution, obviously, against undue damping. An alternative: with an assistant focusing a lamp on the vibrating mass the shadow very nicely traces the curve as the time axis is traveled.

JULIUS SUMNER MILLER

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A Device for Physics Open-House

IT is common practice on American campuses to hold "open-house" sometime in the Spring of the year for the citizenry, at which time departments put on a show of their best demonstrations. The experiments are usually of varied character, some designed to instruct and others to mystify. Some are demonstrated by student performers, some call for audience participation. A very simple arrangement which invites a substantial measure of curiosity (and which, incidently, seems altogether overlooked, presumably because it appears trivial) is the following:

Suspend a weight by a long string. A croquet ball is good, and fish line makes a good supporting string. Exactly under the point of suspension, and on the ground or floor, stand a slender stick upright, such as a short length of broom handle or a very large spike. The problem is to pull the bob aside from the vertical and then to project it so that on its *return* trip it knocks the upright over. *Any* projection designed to do so necessarily starts the bob in an elliptic path and the hoped-for result is impossible to achieve.

Interestingly enough, and this may not be a secret, if the supporting string permits appropriate torsional forces to be utilized the trick can be accomplished, but this is not sporting!

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Exterior Ballistics

LAST fall a spirited discussion arose in my elementary physics class about whether the maximum range of a projectile fired on a flat plain was obtained at angles of elevation greater than, or less then, 45 degrees. The effect of air resistance shifts the position of the maximum, but it is by no means immediately obvious in which direction. In order to answer this question by experiment (and to pose some new ones more in the tradition of the elementary physics laboratory), I planned an expedition to the College athletic fields for the following laboratory period.

The expedition was armed with

(1) an air rifle with several hundred rounds of B-B ammunition;
(2) a 50-foot flexible steel rule; and
(3) a machinist's protractor with a built-in spirit level and a meter stick which could be used to extend the length of its rule.

I deployed my forces so that at least one person would have a chance of seeing where the first BB shot landed. It is necessary to effect a compromise between the requirement of student safety—the spent BB might just conceivably hit someone in the eye—and the need to be rather close to the point where the pellet lands; it is rather hard to follow a BB in flight. The watchers then stood about seven feet to the side of the expected landing place; they did not face the sun. One person fired the gun at right angles to the wind direction while another measured the angle of elevation with the protractor-meter stick arrangement. A measure of the reproducibility of this angle, and of the muzzle velocity and aerodynamic characteristics of different BB's, is that at an angle of 12° four pellets, fired by two different people, fell within 10 ft of one another, with an average range of 174 ft. The students put down a marker where each BB landed, and measured to the center of gravity of the markers. A range of 275 feet was found at 30°. Unfortunately, at this point the spring lock mechanism of the air rifle refused to hold any longer, so data were obtained only for two values of the angle of elevation and the question of angle for maximum range was answered only indirectly.

The class returned (reluctantly, I may add!) to the laboratory, where each member computed the muzzle velocity of the gun using the familiar Galilean equations and the simplifying assumption that the horizontal level of the shot is the same when it is fired as when it lands. For the lower angle the apparent muzzle velocity turned out to be about 117 ft sec^{-1}, for the higher about 101 ft sec^{-1}. This discrepancy illustrates the fact that *the equations used do not represent the physical situation*. It can be pointed out that results such as these indicate that the effect of air resistance is to decrease the apparent muzzle velocity at higher angles of elevation (a point which is by no means proved by the scanty data presented here), and so the maximum range should lie below 45°.

We generally illustrate the principles of mechanics in the laboratory with small-scale mock-ups of the physical situations to which mechanics applies; the analogy is often misunderstood by the student. Here is an experiment on the firing of an actual gun; we are putting our theories to test in a reasonably familiar, life-size fashion. The subject of projectile motion is usually discussed in October, so equable weather can be expected. And this is one experiment the students really enjoy.

Improvements naturally suggest themselves: the gun could be bolted to a stand, and the BB made to puncture a large sheet of paper stretched on a wooden frame. But I

suggest that this is the sort of experiment that is better kept in its primitive state. One worthwhile improvement is to use a football field; wind effects can be partly compensated by firing in both directions. The BB pellet could of course be more easily located if it were made to land on a dusty tennis court or parking lot, but it is doubtful that such arrangements can be made at most colleges.

The author is indebted to Miss Nancy J. Scott for her aid in testing the feasibility of this experiment for class use.

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Self-Centered Shadow

ONE of the greatest joys of physics lies in finding explanations for common everyday phenomena which confront all of us, yet which are not quite obvious.

The following example of such a phenomenon excited my curiosity even when I was very young: standing on the end of a diving board over deep water, the sun behind me, I often wondered as I studied my shadow on the water's surface about the rays which seemed to emanate from my head. They radiated from a spot between my eyes, and the rays seemed to glisten and to rotate like the spokes of a wheel, generally rotating continuously in one direction. When a friend would join me, we would both see the rays coming from our own heads. This phenomenon is observed sufficiently seldom to elude investigation, for it is not a very striking effect and requires:

1. deep water; over three feet, depending on the nature of the bottom
2. sunshine
3. suspended particles, such as algae or minerals
4. relatively quiet water.

The explanation is straightforward, though very satisfying: Sunlight produces the same effect in water containing suspended particles as it does in air, where it produces a sunbeam. In the phenomenon described above, the sun, observer's eyes, and shadow comprise the optic axis. Rays of sunlight strike the water parallel to this axis and, if the water be unperturbed, continue through the water parallel to each other. If, however, the surface be disturbed by superposed wave trains, traveling this way and that, the regions where the rays are refracted and continue as through still water (i.e., level patches of water) become spaced according to the surface wave form.

Thus shafts of sunlight within the water are the result of a ruffled surface. An observer looking down into the water sees these shafts but, because of his viewpoint on the optic axis, they appear to radiate from a common center in much the same manner as the lesser buildings appear to radiate from an observer looking down from atop the Empire State Building.

As for the apparent rotation of the "spokes," this is due to the effect of the moving wave trains. Although the directions of the wave trains may differ, there is generally uniformity in the sense of their travel. As the wave trains progress, they produce the observed shifting in the shafts, causing an appearance of rotation.

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AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 3 MARCH, 1953

ANNOUNCEMENTS AND NEWS

Cloud Chamber Photographs of the Cosmic Radiation. G. D. ROCHESTER AND J. G. WILSON. Pp. 128+vii, Plates 123, 21×27.5 cm. Academic Press, Inc., New York, 1952. Price $10.80.

Few instruments have played as long and as distinguished a role in research in unraveling the mysteries of fundamental particles as the cloud chamber. From the time of its invention by C. T. R. Wilson 41 years ago to the present, it probably has more achievement to its credit of a fundamental nature than any other single instrument. Its chief merit lies in the fact that visual evidence of what occurs can be photographed and the details studied at leisure. As applied to problems of current interest in high energy physics, namely, the study of unstable particles, it has not only been the means by which most of the new particles have been discovered, but it permits a determination of their masses, their energies of disintegration and in some cases their lifetimes.

The book under review was compiled by two research workers who are eminently suited for the task. Both Rochester and Wilson learned the cloud-chamber art as students of P. M. S. Blackett, while Rochester is one of the co-discoverers of V-particles. The word "compiled" is used because the major portion of the 128 pages is taken up with photographs which have been collected from some 20 laboratories throughout the world. Of the estimated 10^6 (by Professor C. D. Anderson) photographs that have been taken by these laboratories, the 123 here selected certainly represent the richest of the cream. The reproductions are excellent and the authors and editors are to be congratulated on the quality of the work.

The topics selected include photographs showing typical methods of operating cloud chambers such as delayed expansion, counter controlled and random expansion. Several historical photographs are reproduced, such as the first published recognizable cosmic-ray tracks taken by Skobelzyn and the positive electron track of Anderson. Examples of cascade showers which are initiated by photons and electrons include their development and absorption in passing through successive plates in the chamber. These latter electromagnetic interactions, which are due to radiative collisions between photons or electrons

with nuclei, seem to be well explained and hold little interest at the present time for those working in the field of high energy particle physics.

Collisions of nucleons causing nuclear disintegrations are given considerable attention. Many of these collisions give rise to an extremely numerous and complex progeny. The authors have done exceedingly well in pointing out the events that have a reasonable explanation and those that have not. In such nuclear encounters one usually can identify protons, π-mesons and electrons. The latter probably originate from the disintegration of π^0 mesons. In addition to these particles it is now believed that many of the secondaries resulting from a very high energy nucleon ($\sim 10^{13}$ ev) may be κ-mesons with a mass of about 1200 electron masses.

The last 20 plates give the best available evidence for the so-called V-particles, both uncharged and charged. In studying these particles the cloud chamber is ideally suited since their lifetime ($\sim 10^{-10}$ sec) is such that they stand a good chance of decaying in the chamber if they are formed in the upper part or immediately above. Much attention is now being given to these unstable particles including τ-mesons, κ-mesons, etc. At the present writing hardly a month goes by without discussion of the possibility of the existence of new particles.

The situation at present is a far cry from that which existed 25 years ago when the material world was thought to be made up of electrons and protons. Ever since the beginning of the century, for those who have had their scientific ear to the ground, there has been little time for complacency. No one realizes better than the theoretical physicist the present state of confusion regarding these new particles and the nucleus from which they appear to come. Edward Teller's statement of this, three years ago, is as true today as it was then, when he opined:

From mesons all manner of forces you get;
The infinite part you may simply forget.
The divergence is large, the divergence is small:
In meson field quanta there is no sense at all.
What, no sense at all?
No, no sense at all.
Or, if there's some sense, it's exceedingly small.*

H. V. NEHER
California Institute of Technology

*Edward Teller, "The Meson Song," Proceedings of the Echo Lake Cosmic-Ray Symposium, June, 1949.

Theoretical Nuclear Physics. JOHN M. BLATT AND VICTOR F. WEISSKOPF. Pp. 864+xiv, Figs. 117, 15×23 cm. John Wiley and Sons, Inc., New York, 1952. Price $12.50.

This long-awaited book presents conventional nuclear physics for graduate students and for researchers who desire a reliable and intelligible source book. Almost all phases of the science are touched upon but particular attention is given to the interpretation of nucleon-nucleon interactions, nuclear stability and spectroscopy, theory of nuclear reactions and radiations alpha, beta, and gamma. As is well known, the authors have contributed previously to theoretical research in most of these fields.

The subject matter is presented very clearly; every effort is made to appeal to the reader's intuition and great care is taken in the presentation of formulas. The usual difficulty of finding enough symbols for the latter has been resolved by requiring consistent usage only within a given chapter and appending a glossary to each. This accounts for 100 of the 800-odd pages of the text. There are over 120 figures depicting relevant experimental data and theoretical calculations but mainly illustrative sketches similar to those put on the blackboard in classroom presentation. In fact, the tone of the book is essentially that of a collection of lectures. Nothing worth saying is said less than three times. It is this method of writing and clearing up such mysteries as tensor forces, supermultiplets, effective range, resonance reactions, scattering matrix, forbidden beta-decay, multipole radiation, and a number of other recondite matters that account for the extraordinary length of the book. Even at that, such explicit treatment can be carried only so far and the authors consistently tie up the loose ends by numerous references to the literature and by promises to pick up the subject in a subsequent chapter. They devote an acceptable amount of space to detailed comparison of the theory with experimental results.

For the most part the text adheres to the conventional vocabulary and usage of research papers in physics. There is an exception, however, in emphasizing *nuclear* masses, rather than atomic masses, and leaving the uninitiated alone in that forest. Some of the developments in the presentation are new but at all points the reader is cautioned when these, as well as classical approaches, are provisional. The contents include an up-to-date chapter on general properties, an exhaustive semi-empirical analysis of nucleon-nucleon interactions, and 250 pages on theory of nuclear reactions ranging from the most elementary concepts to the formal developments of Wigner and Eisenbud. The theory of the ground-states of nuclei is divided into a short chapter on three- and four-body problems and one on nuclear shell structure, a rather extensive treatment of saturation and effect of symmetry, especially Wigner's theory of the symmetric Hamiltonian, and analyses of mass formulas and nuclear models. Alpha-decay is presented in a short chapter whereas large chapters are devoted to beta- and electromagnetic radiations. The latter contains many formulas that have not been published, at least not in collected form. The book has very few typographical errors.

The authors purposely omit extensive discussion of subjects not concerned with the nucleus proper. This excludes theory of ionization loss, diffraction, polarization, slowing down, etc., of neutrons, theories of radiofrequency and hyperfine techniques in nuclear physics, nuclear phenomena in cosmic rays, chain reactions, and the attempts to relate nuclear phenomena to meson field theories. Moreover, there is no consideration of the problems of stellar energy nor of the formation of the chemical elements. In brief, the book is devoted to bringing the reader up to date on what is widely accepted about the nucleus itself,

without building up the generality of the theories; and in no place does it betray the reader and lapse into erudition. It is written for the "beginner" and includes appendices on quantum mechanics and radiation. With it as a basis the student can readily pursue the subjects listed as omissions above and also amplifications of topics such as density of levels, Coulomb wave functions, angular distribution, and polarization in reactions that are not developed so exhaustively. To such and others that desire introduction to and information on the subject matter included I recommend the book wholeheartedly.

CHARLES L. CRITCHFIELD
University of Minnesota

High-Energy Particles. BRUNO ROSSI. Pp. 569+xii, Figs. 245, 15×23 cm. Prentice-Hall, Inc., New York, 1952. Price $12.50.

High energy physics, as defined by Professor Rossi, deals with particles of energy greater than 10 Mev. The study of such particles was once the exclusive province of cosmic-ray physicists, but with the development of high energy accelerators, the opportunities for more controlled and precise experiments developed rapidly and there is available today an enormous amount of data on high energy particles. Rossi has undertaken to present the most important experimental data, together with many necessary theoretical developments in a book suitable as an advanced textbook and for reference and review. He has succeeded admirably in a most arduous task. There are, for example, over 500 references quoted, many in detail, two-thirds of which have been published since 1945. The important facts discovered in each experiment are presented with clarity and precision, in a form designed for quick reference, but preserving a logical, even historical approach.

In the theoretical sections of the book, principally on electromagnetic interactions and cascade shower theory, the physical principles of electromagnetic theory form a solid foundation for the mathematical developments. Rossi has omitted meson theory since it has so far proved to have limited usefulness in the interpretation of the results of experiments. Electromagnetic theory, on the other hand, is essential, and the many graphs, tables, and equations relating energy loss, momentum, scattering, range, and the many other quantities that the high energy physicist uses every day will be most useful. The cascade shower theory is treated in considerable detail, and is perhaps somewhat out of proportion to the rest of the book, from the point of view of the experimenter who never quite knows what shower theory to believe and thus hesitates to use any.

In the chapter on experimental methods, the various devices used in the detection of high energy particles are discussed and compared critically, and their behavior and characteristics are summarized in tables of data and graphs. The history of the discovery of the various types of mesons is given in a chapter on elementary particles, and there is also a chapter on the results of experiments on the electromagnetic interactions of fast charged particles, including some of the classic experiments by Rossi himself. The last two chapters contain accounts of many important cosmic-ray and accelerator experiments on the properties of fundamental particles. It seems clear that cosmic-ray research and accelerator research are converging and overlapping fields, but often the workers in the two areas do not speak quite the same language. Thus the tendency for the two methods of research to remain separate because of differences in technique and outlook is accentuated by misunderstandings in terminology. Rossi is always careful to define the terms he uses, especially in connection with cosmic rays where certain words are often used carelessly. In these last two chapters on nuclear interactions Rossi demonstrates convincingly the unity of high energy physics. He has included dozens of beautiful emulsion and cloud-chamber pictures which vividly bring to mind the complexity of the events and the difficulty of interpretation.

A book that will be as useful as this one deserves to be well made, and it is. It is an expensive book, but it will be read widely and promises to become the constant companion of high energy physicists. For nonspecialists it provides a valuable summary of the results in an important and fast-moving field of physics.

WILLIAM B. FRETTER
University of California

Radio Astronomy. BERNARD LOVELL AND J. A. CLEGG. Pp. 238, Figs. 120, 14×18 cm. John Wiley and Sons, Inc., New York, 1952. Price $4.00.

The book is easily written and easily read. It is an excellent book for a person who wishes to gain a quick understanding of what has been learned (up to about 1951) in the new and rapidly expanding field of radio astronomy. The first third of the book gives an elementary review of astronomy and of radio waves and radar. The second third describes the radio observations of meteors. This is the best and the most authoritative part of the book because the authors have directed the work at the Jodrell Bank Experimental Station of the University of Manchester, England, which has been concerned mainly with the investigations of meteors by means of radio echoes from the meteors themselves and from their tracks of ionization in the upper atmosphere.

The final third of the book deals with radio emissions from the sun and from the radio stars, with brief discussions of radio echoes from the moon and from the aurorae. These subjects are really radio astronomy, and are the ones which are being most vigorously pursued at the present time. The "letter to the editor" of yesterday is the expanded work of today of several institutions. Many of the statements in the book already require revision. A surprising omission is that nothing is said of the important results obtained by Covington in Canada and by workers in this country from continuous records since about 1947 of the solar emissions on 3, 10, 60, and 180-centimeter waves. Already nearly 80 radio stars have been discovered. The authors ask, What are the radio stars? Are they new types of stellar objects, or are they stars in the process of being born which are not yet hot enough to be seen by visible light? What are the distances of the radio stars?

The answers to these questions are not yet known, although four of the radio stars appear to be associated with fast moving nebulae.

EDWARD O. HULBURT
Naval Research Laboratory

Symbolic Logic, An Introduction. FREDERIC BRENTON FITCH. Pp. 238+x. The Ronald Press Company, New York, 1952. Price $4.50.

Mr. Fitch, who is professor of philosophy at Yale, offers a textbook in symbolic logic which is quite orthodox in calculative devotion but has several newnesses both of presentation and theory. The chief newness of theory is probably the way proposed as escape from the theory of types, but that is beyond the scope of this note.

Mr. Fitch says (page v) of his logic that it is a system that "seems to be adequate for all mathematics essential to the natural sciences." He notes (pp. 3–5), "There have already been applications of symbolic logic to problems in biology, neurophysiology, engineering, psychology, and philosophy," with references. He finds the chief future value of symbolic logic in its giving "possession of a tool that is powerful enough to be of help in reasoning about relations and qualities of all sorts," not "just the familiar numerical and geometrical relations of mathematics, together with the relation of identity and some relations of implication and class-inclusion," so that "the day may come when it will be as improper to study ethics and politics without a thorough grounding in symbolic logic as it now is to study physics without a thorough grounding in mathematics." And "we should not expect the logic suitable for philosophy, ethics, and esthetics to be very much less technical than the mathematics suitable for physics and chemistry."

The relation of the "new" logic to physics may be a test case. I take it as generally agreed that physics, for all its debt and its loyalty to mathematics, is more than mathematics (and is not all of mathematics). And this is so, not only in that physics has a concrete subject matter to which mathematics is applied, but also in that application is not just a mathematical process or mathematics-plus-observation but uses some methods traditionally called logic, or common sense, or reasoning. In so far as these operations are upon "relations" other than the quantitative ones, it would seem that the physicist is in the same position as the ethical and political theorists Mr. Fitch names and that symbolic logic can be expected to work alongside mathematics as it works, perhaps alone, in, say, ethics. And I do not think it can be denied that symbolic logic is workable in this way in physics (and elsewhere); but two questions arise: can the present, or the present sort of, or any sort of symbolic logic do everything in physics which could be called logic and which is not conventional mathematics; and is it in the ideal of Mr. Fitch and the more extreme party that symbolic logic be expected to take over the tasks of conventional mathematics also? Since one chief aim, as noted by Mr. Fitch, is to give a basis for mathematics and because of the symbolists' characteristic method of ignoring distinctions by appeal to abstractive similitudes, it would seem symbolic logic must come to a still more abstract and simulitudinous system underlying both the "familiar" quantitative relations of mathematics and those others of the nonquantitative subject matters. This should allow a metamathematical treatment of some parts of physics which hitherto have seemed aside from mathematics; but it might be a treatment less competent than the older logic for the concrete and judgemental tasks of the physicist, as well as less competent than mathematics for the more abstract but still not purely symbolic work hitherto done by mathematics. Doubtless it will be argued that mathematics may be left as a particularized calculus to do the jobs to which it has become accustomed and a logic, made more abstractive by the symbolists' criticism, shall do the rest, with the subcellar of pure and universal abstraction and nondistinction left for the theorists of the theory of mathematico-logic. In this situation common sense itself might take heart to ask a chance to serve the human physicist in some tasks not refined enough for either mathematics or symbolic logic, perhaps also to supply a study of judgement so that the physicist may decide when to use which of these tools.

Mr. Hans Reichenbach has recognized the need for something more than mathematics or than what is common to mathematics and logic by adding to his "adjunctive" (material implication) logic a "connective" one for ordinary language whether in physics or in politics. Mr. Fitch ignores any such way out and holds, perhaps more consistently, to the symbolic highway.

Mr. Fitch, as we have seen, foresees moral, artistic, and political writers using and needing this logic as well as the formal and natural scientists. Is it to be taught as all of logic? Can all students be taught it? Teachers give curiously different testimony. Perhaps the physics teacher, if interested, since he is apt not to be as prejudiced, can come to better opinion than the logic teacher. Can all students of physics be taught symbolic logic and is this all the logic any will need to be taught or will need to use?

If on the one hand we accept the view that this is all of logic and on the other accept the view that here if anywhere is specialized ability—that some students take to it, and some can take it or leave it, and some have to leave it; the middle class being vanishing—then we would seem to be heading for a curious outcome for the dream of Bacon and Descartes, the dream of a method which, like the compass for the hand, should level all thinking up and end the call for native genius in science. Today we seem to approach a super Solomon's House wherein at the top will be a very few symbolic masters, the devisers and critics of the system. Below them a larger but still inevitably small number will contrive the mechanical brains; and a larger number will direct and feed and interpret the machines. There will then be the army of machines to do the actual work of applied thinking. But the mass of mankind will be outside, absolved from all thinking and debarred by natural incapacity.

ALBERT L. HAMMOND
The Johns Hopkins University

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 21, 72 (1953)].

Anderson, Elmer Ebert, Deep Springs, Calif.
Anderson, Milo Vernette, 4532 Calvert, Lincoln 6, Neb.
Ansley, William Gene (*J*), R. R. 2, Montpelier, Ohio
Batch, Melvin L. (*J*), 2023 Kelsey Avenue, Toledo 5, Ohio
Bernarda, Sister Mary, Regis College, Weston 93, Mass.
Boekelheide, Irving, Montana State University, Missoula, Mont.
Burkacki, Francis Anthony (*J*), 3148 Salmon Street, Philadelphia, Pa.
Cameron, Frank Maris (*J*), Box 213, Emory University, Ga.
Dankert, Wilbur Leroy (*J*), 9 Vandercook Hall, Western Michigan College, Kalamazoo, Mich.
Dewdney, John Wells, 37 Buelah Avenue, Hamilton, Ontario, Canada
Elkin, Sanford (*J*), 1115 E. 165 Street, Bronx 59, N. Y.
Escobar, Ismael Vallejo, Presbitero Meolina, La Paz, Bolivia
Foster, Howard Jerry (*J*), Box 156, Fisk University, Nashville, Tenn.
Gallagher, Tom Lewis (*J*), 102 Avenue B, Denton, Texas
Goldstein, Raymond (*J*), 1528 Overing Street, Bronx 61, N. Y.
Green, Robert Boyce, Department of Physics, Stevens Institute of Technology, Hoboken, N. J.
Hecht, Karl, c/o Messrs. E. Leybold's, Nachfolger, Bonner Strasse 504, Köln-Bayental, Germany
Hinds, Conrade Carlyle (*J*), Box 156, Fisk University, Nashville 8, Tenn.
Irvine, Merle M., 35D E. Laurel Street, Bethlehem, Pa.
Jarnagin, William Spencer, c/o Texas Wesleyan College, Fort Worth, Texas
Jennings, Robert Lee, Box 401, State Teachers College, Fayetteville, N. C.
Jeppesen, C. Rulon, Montana State University, Missoula, Mont.
Jones, Robert William, Lowell Textile Institute Research Foundation, Lowell, Mass.
Marchant, John Carl (*J*), 12E Vet Village, Oxford, Ohio
Massingill, George Shaffer, P. O. Box 574, Corsicana, Texas
McCray, James Arthur (*J*), 886 S. Belmont, Decatur, Ill.
McCormick, Philip Thomas, 559 S. Kenilworth, Elmhurst, Ill.
McManus, Vincent Joseph (*J*), 27 E. Pastorius, Philadelphia 44, Pa.
Mitchell, J. Taylor, 1010 Douglas Street, Las Vegas, N. M.
Molmud, Paul, Apt. 26C, C.C.H.P., Potsdam, N. Y.
Morrison, Robert Eugene, c/o Severn School, Severna Park, Md.
Packard, Robert Gay, Box 287, Department of Physics, Baylor University, Waco, Texas
Parsons, Karl A., Michigan State Normal College, Ypsilanti, Mich.
Puttcamp, Robert Raymond (*J*), 2513 Benton Avenue, Kalamazoo, Mich.
Rank, Ronald I., 606 Jackson Street, Prophetstown, Ill.
Rosen, Arthur Zelig, University of California, Santa Barbara College, Santa Barbara, Calif.
Sheets, Nicholas J., 440 Windsor Street, Reading, Pa.
Stanley, Robert W., Department of Physics, Clarkson College of Technology, Potsdam, N. Y.
Stoeffler, John Anthony (*J*), 6900 Castor Avenue, Philadelphia 24, Pa.
Wylie, Douglas Wilson, 12-B South Apartments, Orono, Maine
Zei, Dino, 743-B College Street, Beloit, Wis.

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 3 MARCH, 1953

RECENT MEETINGS

Kentucky Section

The fall meeting of the Kentucky Section, American Association of Physics Teachers, was held on October 18, 1952 in the Science Building, Eastern State College, Richmond, Kentucky. The meeting was attended by 31 members and guests. Professor PAUL C. OVERSTREET, *Morehead State College*, presided. Following the morning session, a business meeting was held during the lunch hour. The contributed papers were as follows:

1. A low-pressure ultrasonic condenser microphone. CARL E. ADAMS, *University of Louisville*.

2. An extrapolation chamber for determining beta-ray surface-dosage rates. JOHN M. BRABANT, *University of Kentucky*.—An extrapolation ionization chamber method of determining the dosage-rate at the surface of beta-radioactive sources used in the treatment of certain eye diseases is discussed. All irradiated surfaces of the chamber were made of tissue-equivalent material; this avoided certain corrections as the sources were then measured in virtually the same physical environment as that in which they were used. The dosage rates were evaluated by extrapolating the ionizing volume to zero. The correction for the absorber between the source and the measuring volume was made by extrapolating a dosage-rate *vs* absorber thickness curve to zero absorber. As the various extrapolations eliminated the effect of the measuring device upon the quantity being measured and the measurements do not require the use of a known source, the method is seen to be an absolute one. Results and plans for im-

provement are discussed. Present reproducibility is better than five percent.

3. Spectroscopic measurements with the Echelle. R. A. LORING, *University of Louisville.*

4. A modern physics laboratory course involving construction techniques. DONALD C. WORTH, *Berea College.*—This was described as a one- or two-hour, two-semester, laboratory course. The experiments themselves are intended as appropriate and approximate duplications of some of the "landmark" discoveries of modern physics. The feature which is perhaps unusual is that the student spends a good part of the first semester acquiring some facility in machining, soldering, glass-working, photographic, vacuum system, and other techniques. These are intended, in part, to aid him in the design and construction of one new piece of apparatus for subsequent use as one of the course experiments.

The objectives of the techniques portion of the course were presented as: (1) To teach some important construction and laboratory techniques: (2) To illustrate the role of developmental techniques in research: (3) To accumulate future standard experiments and equipment.

With regard to these objectives, some advantages and experiences were discussed. These aspects of the course can give a wider perspective to the student than the usual laboratory-arts course and can give the student at the same time specific incentives toward the completion of his apparatus.

Presentation of the specific laboratory techniques incorporated in the course followed, accompanied by some group discussion.

5. The need and place of a foreign language for a physics major or an engineer. EARLAND RITCHIE, *Centre College.*

RICHARD HANAU, *Secretary-Treasurer*

Southern California Section

The annual fall meeting of the Southern California Section of the American Association of Physics Teachers was held at the California Institute of Technology on Saturday, October 4, 1952. About one hundred members and friends enjoyed the day's program. It began with an invited paper, **The long range implications of atomic warfare** by DR. ALBERT W. BELLAMY, Chief of Radiological Services, California State Office of Civil Defense. The remainder of the morning was devoted to a talk on the synchrotron of the California Institute by DR. ROBERT WALKER and a visit to the synchrotron.

After luncheon at the Athenaeum, during which the group was welcomed to the California Institute of Technology by DR. LEE A. DUBRIDGE, the group returned for the afternoon program of contributed papers. These were as follows:

1. Education?—or merely training? XI Swivel-chair or pipe-dream laboratory techniques. GEORGE FORSTER, *Pasadena City College.*—The instructions for an experiment designed to test Charles' law of thermal expansion of a gas led to such poor results that one is compelled to question whether or not the author of the instructions wrote them purely from imagination and without testing them before publication. The reason for poor results is that the effects of water vapor pressure were not considered. To obviate the difficulty the procedure has been modified as follows. An elongated glass bulb with a capillary tube opening at one end is heated in a double-walled hypsometer, then capped with a rubber "policeman," cooled to 0°C in an ice bath, and uncapped temporarily to allow water to replace the air lost during the expansion. From temperature measurements and the mass of the bulb obtained (1) with dry air, (2) when partially filled with water, and (3) when completely filled with water, the change in volume and the coefficient of expansion of the air at constant pressure are determined. The results are very gratifying, and together with a number of other values offered in the modified instructions make the experiment very worthwhile.

2. The expansion of a rotating wheel. LESTER L. SKOLIL, *San Diego State College,* AND DONALD A. NORTON, *University of California at Davis.*

3. A project approach for the elementary laboratory, II. BURTON HENKE, *Pomona College.*—A project approach for the elementary physics laboratory has been developed through the past four years at Pomona College. This kind of laboratory has been very stimulating to student interest and initiative, and has provided a realistic educational experience with the experimental method. The experiments employed are those often used in the traditional laboratory. However, each is presented as a problem, usually with certain pertinent questions to be answered, and no written instructions as to method are given. To permit the project team enough time to discover appropriate methods of solution, two weeks rather than one are allowed for each experiment. New ideas are presented by frequent discussions both among the students and with the laboratory assistants. (The laboratory assistants are physics majors, usually juniors and seniors, who meet weekly with the instructor to discuss the various projects.)

The experiments are presented so as to teach certain techniques of the experimental method, along with the physical principles. Usually the problems involve the determination of empirical equations which describe the phenomena observed. Dimensional analysis, log-log plotting, along with the usual methods, are taught in this connection. The significance of errors and the evaluation of probable error are demonstrated in an experimentally derived error curve which the team obtains from a dart game.

4. Physics teaching in Baghdad. ALBERT B. BAEZ, *University of Redlands.*—The College of Arts and Science of Baghdad, Iraq, has established a physics department with the aid of UNESCO Technical Assistance. Aid in the form of personnel, laboratory equipment and facilities, library acquisitions, and a planned curriculum have been extended.

5. The place of theoretical physics in the undergraduate curriculum. JACK C. MILLER, *Pomona College.*—In the areas of study between mathematics and physics two different forms of theory are recognizable. The first of these is what may be called mathematical physics and

deals with the tools of use to theoretical physics, while the second, theoretical physics, is primarily concerned with physical theory as such. Both of these areas of study offer an opportunity to interrelate the mathematics and physics of the undergraduate curriculum. The requisites for a more extensive program of theoretical physics in the undergraduate curriculum were discussed, and a proposal for closer integration between mathematics and physics together with as much tutorial type of instruction as possible was made.

6. Teaching physics 1900 to 1906. W. P. BOYNTON, *Whittier, California.*—An earlier critical period, that from 1890 to 1893, saw the initiation of the system of tax-supported high schools in California, ultimately relieving schools of collegiate rank from the necessity of maintaining their own preparatory departments. The same period saw the flowering of the new University of Chicago, and the opening of Stanford University, and of Throop University, now better known as the California Institute of Technology. The opening years of the century heralded the expansion of educational institutions to their present overwhelming magnitude. Contrasted are the simplicity of the earlier days, and the wealth, crowded conditions, and red tape of the present.

7. A comparison of the just scale and the scale of equal temperament. ARCHIE WADE, JR., *Los Angeles Conservatory of Music* (Introduced by WILLARD GEER).

8. Two interesting problems and a proposal for stimulating young physics teachers and for inducing them to remain in teaching. JULIUS SUMNER MILLER, *Ford Foundation Fellow, University of California at Los Angeles.*—(1) The problem of the coiled hose. After using a fifty-foot length of half-inch plastic garden hose (you can see the water motion in it very clearly) I rolled it up into a coil of radius two feet or so and thus trapped water in all the loops. Both open ends were at the top of the vertical coil and the loops were full of water. At the moment of hanging the hose on a hook about five feet from the ground one open end fell off, whereupon the water at once flowed out of this straight element and subsequent siphon action drew upon the water in successive loops. When this siphon action ceased water remained in the lowermost arcs of the coiled hose to the extent of roughly one-quarter of the lower half-coils. The question which at once came to mind was: can siphon action empty these coils completely? This would seem to be accomplished by having the efflux end far enough below the lowermost portion of the coils, and so I repeated the inquiry each time allowing the open end to fall farther and farther below the coils. How did matters finally stand?

(2) The problem of the highly viscous oil. A certain brand of baby oil comes in large 12-oz. bottles and small 4-oz. bottles. The larger is more economical but the smaller is easier to handle. Both bottles are equipped with *small* orifices in the neck of the bottle so that oil is gotten out in drops by shaking the bottle as one would do with hair tonic. The oil will not run freely however the bottle is held, nor will it come out even in drops unless the bottle is shaken. If, now, we have a large bottle full of oil and a small one empty, how can the oil be *readily* transferred from the large bottle to the small one?

(3) The American Association of Physics Teachers now gives very notable recognition for teaching performance in the Oersted Medal. The spirit of this award is laudable, but in general it goes to men *who have taught*. I believe we should offer something to the young men who need and deserve a little urging to make teaching their career. All of us who have been at the game some time know at least one young man who should *now* be recognized. If he is not yet formally in teaching a little special notice or commendation would give him a very spirited boost; if he is already in the profession such recognition will raise his sights and lift his spirit. He may even aspire to the Oersted Medal! I come now to the crux of the matter. Can we not establish some award (or reward) for the men I have in mind? Maybe an honor roll would do it. Maybe an expense-free trip to the New York Meeting of the Association. Who wishes to nominate the first recipient of this recognition?

At the ensuing business meeting one of the items presented was the report of the Eighth Annual High School Physics Test given May 10, 1952 at four locations in Southern California. Two-hundred-fifty-eight students from sixty-one high schools took the test and on the basis of it scholarships to the California Institute of Technology, University of California at Los Angeles, Whittier College, Pomona College, and the University of Southern California were assigned.

As is our custom, an executive committee meeting concluded the activities of the day. The 1953 Spring Meeting is scheduled for March 14 at the Bureau of Standards in Corona.

DAVID F. BENDER,
Secretary-Treasurer

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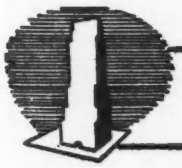

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